

1     **Safety assessment of hydro-generating units using experiments**  
2                             **and grey-entropy correlation analysis**

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20    **Abstract:** This paper focuses on the safety analysis of a nonlinear hydro-generating unit  
21    (HGU) running under different loads. For this purpose, a dynamic balance experiment  
22    implemented on an existing hydropower station in China is considered, to qualitatively  
23    investigate the stability of the system and to obtain the necessary indices for safety  
24    assessment. The experimental data are collected from four on-load units operating at  
25    different working heads including 431m, 434m, 437m, and 440m. A quantitative analysis  
26    on the safety performance of the four units was carried out by employing an integration of  
27    entropy weights method with grey correlation analysis. This assisted in obtaining the safety  
28    degree of each unit, providing the risk prompt to the operation of nonlinear

hydro-generating units. The results confirm that unit 4 has the highest level of safety while unit 3 operates with the lowest safety condition. This provides the optimal operational schedule of HGU's to cope with the fluctuations of electricity demand in the studied station. The proposed methodology in this paper is not only applicable to the HGU's in the studied station but could also be adopted to assess the safety degree of any hydropower facility.

**Keywords:** hydro-generating unit; dynamic balance experiment; safety analysis; grey-entropy correlation;

## 1. Introduction

Renewable energy is unarguably one of the most critical governing factors for today's increasing global economic and social development [1]. The pressing challenge lies in the sustainable harnessing of reliable, secure and affordable energy [2]. To date, hydropower has been the main renewable source of electrical energy for many countries' power consumption (e.g. 99% in Norway, 86% in Brazil and 76% in Switzerland) due to the environmental consequences of fossil fuels exploitation [3]. The electricity provided by hydropower contributes about 16% of the world total electricity generation and is expected to grow to 2 GW in thirty years [4]. It is therefore no exaggeration that hydropower represents more than 92% of generated green energy making it a significant contributor to the global electricity supply [5].

Hydropower stations are the major electricity generation facilities in which the hydro-generating unit (HGU) is the heart of the energy production, transmission and

conversion in each station [6]. HGU is a complex nonlinear system that integrates the characteristics of fluid, machinery, and electromagnetic induction [7]. A universal HGU is comprised of various coupled components such as hydraulic turbines, shafting systems, generators, governors, and excitation systems ([8] to [12]).

Due to the nonlinear coupled characteristics, several hazardous factors are present within the operation of an HGU including shafting vibrations, electromechanical delays, stochastic instability, and inefficient operation. A large number of literatures have extensively studied such topics from the perspective of individual subcomponents, which supports the research foundation for the safety study in this paper. For instance, literatures ([13], [14]) analyzed the cause of shafting vibrations in an HGU. Literature [15] studied a class of hydro-turbine with electromechanical delays. Researchers in ([16], [17]) modelled stochastic variables of an HGU to analyze its effect on the stability of subcomponents. Researchers in ([18], [19]) studied the adaptation strategy of hydropower systems to improve the operating efficiency. This range of conducted research highlights that the hydropower industry is greatly concerned about the safety of HGU operations and improvements are needed [20]. In particular, with the construction of large-capacity hydropower stations to be completed in the following decades, resolving the stability problems of operation, from the perspective of systemic properties, will be one of the major areas that attracts a great deal of attention from the industry [21]. Although a large number of advanced safety assessment methods have been developed in various research fields such as information science [22], ecological engineering [23] and marine engineering [24,

25], the operational safety of HGUs has been rarely investigated and very little evidence of achievements has been previously provided.

To date, the safety analyses of HGUs have mainly focused on investigating the stability of HGU components. The developed methods determine the instability status of the HGU components in terms of narrow hydraulic, mechanical, or electrical angle. However, the integrated safety level of the entire HGU system has not been evaluated from these independent components. Hence, there is a need for a framework that can assess the safety of HGU from the system perspective. Previous researches ([26] to [30]) developed a framework, combining the method of entropy weights and grey correlation theory to investigate the quality problems in different applications such as wastewater treatment, soil detection, and machinery fault. Several studies ([31], [32], and [33]) indicate that the method of entropy weights has a great potential for the assessment of complex systems by measuring the uncertainties of structure indices. The outcome of researches ([34], [35], and [36]) reveal that the grey correlation theory can be adopted for various prediction applications of such complex systems based on incomplete information.

The present paper herein investigates the operational stability of a nonlinear HGU and proposes a methodology for safety assessment of these systems. For this purpose, a dynamic balance experiment is conducted on four HGU units, each with a different working head, in an existing hydropower station in China. The experiment is based on vibration parameter, which is the main risk factor of on-load HGUs. Seventeen indices are extracted to qualitatively assess the operational stability of the units. An effective

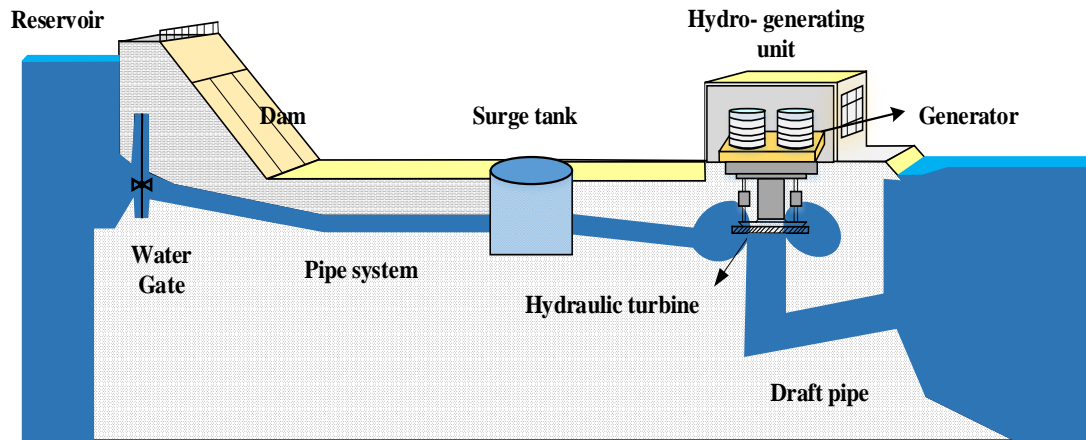
approach integrating the entropy theory and grey correlation is then utilized to quantitatively analyze the safety performance of the studied HGU. This assisted in determining the safety degree of the analyzed four units that run with load, as well as an optimal operational schedule of HGUs coping with peaks and troughs of electricity demand in the studied hydropower station.

The present paper has extensively reviewed the existing literature that are based on the individual subcomponents (e.g. hydro-turbines, shafts and generators) of HGU systems. The major contribution of the paper, however, is to consider the coupled characteristics of hydraulic, mechanical and electrical subcomponents for investigating the safety of HGU operation. Moreover, there are few researches that have successfully applied dynamic safety assessment to nonlinear HGUs. This paper presents a novel methodology that is significantly more flexible and efficient in dynamic safety assessment of HGUs with an attempt to overcome the limitations of static approaches. The safety degree of HGUs is quantified by using a probabilistic approach, which serves as a tool for monitoring and predicting the risk of accidents in hydropower stations resulting from failure in HGUs. This not only improves the safety of HGU operation, but also effectively reduces the operational and maintenance costs of energy production. The results obtained from this research benefit the operators and risk managers of the hydropower industry serving as a tool for development of risk mitigation strategies. For instance, it enables them to respond to the important question of “how to efficiently and safely arrange the operation of multiple HGUs with respect to different allowing heads”.

The remainder of the paper is structured as follows. In Section 2 a brief review of a universal nonlinear HGU is presented. In Section 3 the fundamentals of utilized methods and an overview of the global methodology for safety assessment of HGU are provided. Section 4 discusses the details of the conducted dynamic balance experiment on the studied station's HGU. Section 5 demonstrates the process of safety assessment methodology and presents its highlighted results. Lastly, the key findings of this study are discussed in the conclusion section.

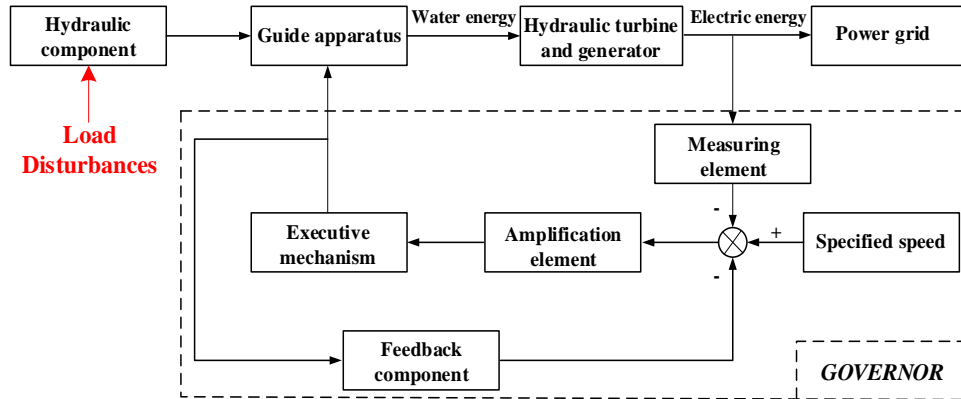
## **2. A Brief Review of an on-load HGU**

HGU is the key equipment of hydropower stations used to produce, transmit and converse electrical energy, which mainly consists of hydraulic turbines, generators, control systems/governors, excitation systems and inlet and draft pipes [37]. The operation of an HGU is always integrated with a number of other hydraulic components such as surge tank, piping system, water gate and reservoir [38]. The structure of an HGU and the key elements of the hydraulic system are shown in Fig. 1.



**Fig. 1** Schematic of an HGU.

HGU, in fact, is a nonlinear system with multi-attribute characteristics including hydraulic, mechanical, electrical and electromagnetic. An on-load HGU is a system synchronized with the power grid, and its load generally cannot be constantly maintained due to the stochastic load. The on-load HGU may be considered as a dynamic system varying with the changes (decrease or increase) in load. An HGU mainly utilizes pressure and momentum energy to produce power. The working mechanism of an on-load HGU is described as the flow velocity influenced by the effect of blade changes as the system load fluctuates, which in turn generates a reactive force in the flow channel. This drives the hydraulic turbines which generate mechanical energy, and the generator further converts the mechanical energy to electrical energy. The details of an HGU working mechanism is presented in Fig. 2.



**Fig. 2** Details of an on-load HGU working mechanism.

In actual hydropower stations, the dynamic performance of HGUs is hard to detect due to the rapid changes in the operational conditions influenced by internal couplings as well as the external environment. Uncontrolled and abrupt changes in the dynamic variables influencing the operational conditions of the system could result in critical damage to the asset as well as other consequences. It is therefore essential to conduct quantitative assessment of the safety and stability of an HGU, probably based on experimental investigations.

### 3. Methodology

Previous researches in this field have focused on developing static safety assessment frameworks for operating HGUs. However, due to the nonlinearity of these systems, attending to the dynamic effects in the analysis are essential for achieving better results. To overcome this shortcoming, an effective method must be developed applicable to hydropower facilities. Through conducting an interdisciplinary research [26, 27], this



section presents the details of an enhanced grey-entropy correlation methodology for dynamic safety analysis of on-load HGUs. The proposed framework is able to improve the imprecision of subjective entropy weights as well as the static evaluation of grey correlation degrees. A major contribution of the established method is in adopting the probabilistic approaches to predict and reflect the real-time safety level of on-load HGUs, which is greatly beneficial when dealing in a timely manner with unexpected accidents and the development of improved safety and risk mitigation strategies.

### **3.1 Entropy Weights Method**

The concept of entropy that is derived from thermodynamics theories represents a measure of disorder in a system. Entropy theory was proposed by Shannon, in 1948, to reflect the uncertainty in information science, it has been applied in various research fields for its precision and flexibility [39].

Two approaches can be applied for determining the weights of indices, known as subjective fixed weight and objective fixed weight methods. Entropy weight method, as an objective approach, is based on the amount of data, overcoming the subjectivity issues as it is independent of expert judgment. The main idea of entropy method is to determine the weights by index variations. In general, a smaller index weight represents a larger degree of index variation, meaning that the index may provide more assessment information and have significant influence on the stability of the system. In the entropy safety assessment of an HGU, a specific index weight is the critical indicator to measure the importance of the selected index, assessing its safety contribution to the studied

178 system.

179 Assuming that there are  $m$  assessment indices and  $n$  assessment units, the assessment  
180 data is transformed into a form of standardization that employs a normalized method of  
181 inverse index, shown in Eq. (1) [40].

$$182 \quad r_{ij} = \frac{\max x_{ij} - x_{ij}}{\max x_{ij} - \min x_{ij}}, \quad i=1,2,\dots,m \text{ and } j=1,2,\dots,n, \quad (1)$$

183 where  $\{r_{ij}\}_{m \times n}$  is the normalized set of inverse index.  $\max x_{ij}$  and  $\min x_{ij}$  are the  
184 maximum and minimum values in the index column of assessment units, respectively. It  
185 should be noted that the lower value of inverse index is most important in ensuring safe  
186 operation of an HGU.

187 Then the entropy value of index  $i$  is determined by Eq. (2).

$$188 \quad E_i = -\frac{\sum_{j=1}^n r_{ij} \ln r_{ij}}{\ln n}, \quad i=1,2,\dots,m \quad (2)$$

189 and the index weight of  $i$  is obtained as:

$$190 \quad \omega_i = \frac{1 - E_i}{\sum_{i=1}^m (1 - E_i)}, \quad \sum_{j=1}^n \omega_i = 1, \quad \omega_i \in [0,1] \quad (3)$$

191 Therefore, the index weight set  $W_i$  is  $[\omega_1, \omega_2, \dots, \omega_n]$ .

### 192 3.2 Grey-entropy Correlation Method

193 Grey system is used to describe an uncertain system that has the characteristic of  
194 partial information loss, and grey correlation theory is a powerful tool to query the quality  
195 of a system with poor information [41]. An on-load HGU is an engineering system

incorporating a degree of uncertainty and therefore it can be assessed by the grey correlation theory. The concept of using grey theory is to find the possible motion rule from the disordered and fuzzy data. Specifically, it is the similarity of an index in different assessment units that is the key factor for measuring the variation between the indices. A greater similarity between indices means that the grey correlation of a studied unit is more optimal. There are no requirements for the size and characteristics of data in a grey correlation analysis which overcomes the shortcomings of traditional regression analyses.

Based on the normalized set of inverse index  $\{r_{ij}\}_{m \times n}$  mentioned in Eq. (1), the index column is expressed as  $x_1, x_2, \dots, x_m$ . It should be noted that, there are  $i$  assessment plans in the analysis, i.e.,  $x_i = [x_i(1), x_i(2), \dots, x_i(n)]$ , where  $x_0$  is assumed to be the optimum plan. Therefore, the correlation coefficient,  $\xi_i(j)$ , between  $x_0$  and  $x_i$  with respect to the  $j^{\text{th}}$  factor in the index set  $\{r_{ij}\}_{m \times n}$  is expressed as [42]:

$$\xi_i(j) = \frac{\min_i(\Delta_i \min) + \rho \max_i(\Delta_i \max)}{\Delta_i + \rho \max_i(\Delta_i \max)}, \quad i=1,2,\dots,m \text{ and } j=1,2,\dots,n, \quad (4)$$

where  $\Delta_i$  is equal to  $|x_0(j) - x_i(j)|$ ,  $\rho$  is the resolution coefficient that changes within the interval  $[0, 1]$ , but generally it is set at 0.5.  $\Delta_i \min$  and  $\Delta_i \max$  denote the minimum and maximum differences in the first level respectively, while  $\min_i(\Delta_i \min)$  and  $\max_i(\Delta_i \max)$  are the minimum and maximum differences in the second level, respectively. The expressions for each of these terms are shown as follows:

$$\begin{cases} \Delta_i \min = \min_j |x_0(j) - x_i(j)| \\ \Delta_i \max = \max_j |x_0(j) - x_i(j)| \end{cases} \quad (5)$$

and

$$\begin{cases} \min_i(\Delta_i \min) = \min_i \min_j |x_0(j) - x_i(j)| \\ \max_i(\Delta_i \max) = \max_i \max_j |x_0(j) - x_i(j)| \end{cases}, \quad (6)$$

Subsequently, based on the index weight  $W_i$  obtained using Eq. (3), we estimate the correlation coefficient  $\xi_i(j)$  for the  $i^{\text{th}}$  studied unit to obtain its integrating safety degree. Therefore, the grey correlation degree,  $\alpha_i$ , between the optimum unit and the studied unit  $i$  is given by the grey-entropy correlation equation as follows:

$$\alpha_i = \sum_{j=1}^m W_i \xi_i(j), \quad 0 \leq \alpha_i \leq 1. \quad (7)$$

In Eq. (7), the obtained grey correlation degree  $\alpha_i$ , also defined as the safety degree, assists in assessing the safety level of a multi-unit HGU from a probabilistic point of view. That is, a higher value of  $\alpha_i$  corresponds to a safer HGU thus for instance, a system with  $\alpha_i = 1$  has the maximum level of reliability.

### 3.3 Global Methodology

This paper presents a novel framework for the dynamic safety assessment of HGUs by combining the entropy weight method with the grey correlation analysis. The major novel components of the proposed method consist of:- firstly, the method overcomes the subjectivity of traditional methods in determining the weight coefficients of assessment indices, which improves the accuracy of the results and provides a more scientific

representation. Secondly, the method completely transforms the static safety assessment into a dynamic practice by substituting the dynamic entropy weights (i.e. Eq. (3)) into the relationship for obtaining the grey correlation degree (i.e. Eq. (7)). Thirdly, few existing studies have been proven to be successful in conducting a probabilistic safety analysis of nonlinear HGUs.

The steps of the developed methodology in this paper are provided in Fig. 3, and summarized as follows.

(1) A dynamic balance experiment is carried out on the existing HGUs for different allowing heads, to qualitatively analyze the dynamic operational behavior of a hydropower station. The obtained data,  $m$  assessment indices for  $n$  studied HGUs, is later used to conduct a quantitative safety analysis.

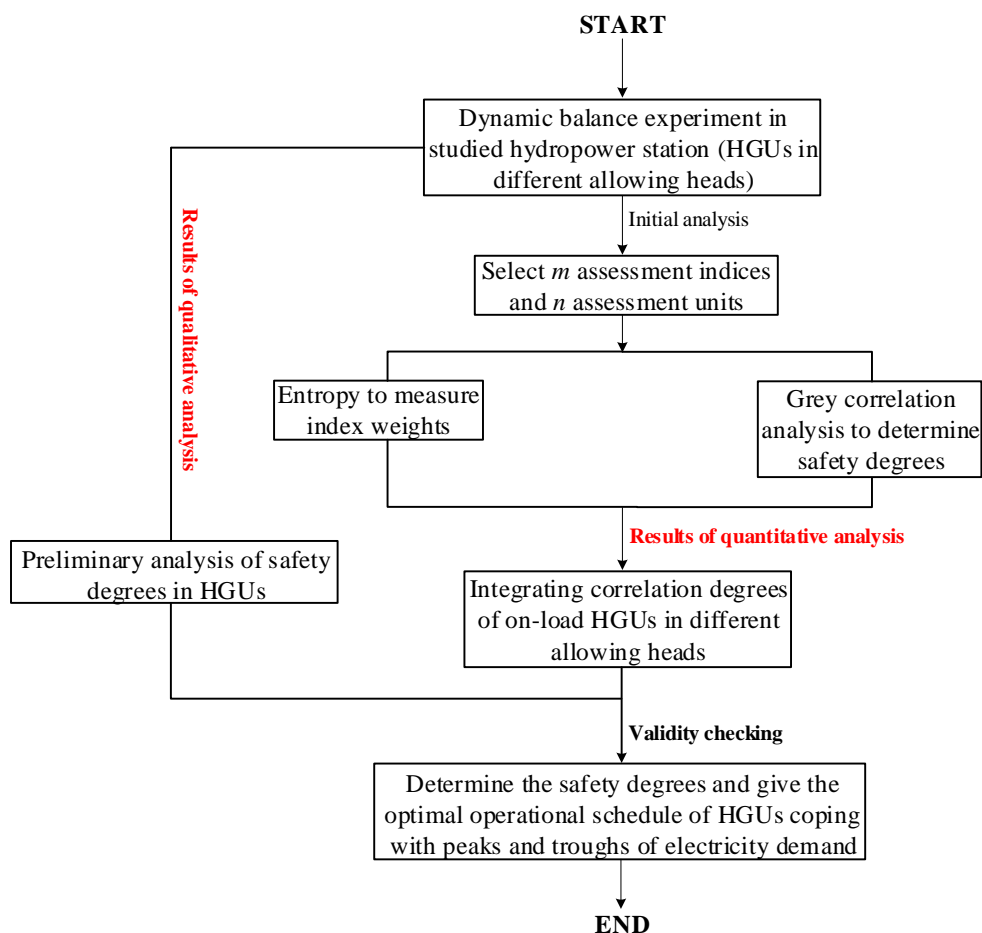
(2) Dynamic entropy weights (see Eq. (3)) are developed to estimate the contribution of the indices on HGSSs' stability with respect to time. For this purpose, the indices with significant influence on HGS' operation under various allowing heads are identified.

(3) The grey-entropy correlation degrees (see Eq. (7)), combined with the dynamic entropy weights (see Eq. (3)) and grey correlation coefficients (see Eq. (4)), are used to evaluate the safety degree of  $n$  studied HGUs. The safety degree is expressed by probability values.

(4) Based on the quantitative analysis, the time-varying safety state of HGUs and any accidents are revealed. This enables the technicians and operators of hydropower stations to make an optimal operational schedule of HGUs for dealing with fluctuations of

electricity generation and demand.

A detailed illustration of the numerical process of entropy weights and safety degrees is presented in the Appendix.



**Fig. 3** Proposed framework for safety assessment of on-load HGUs.

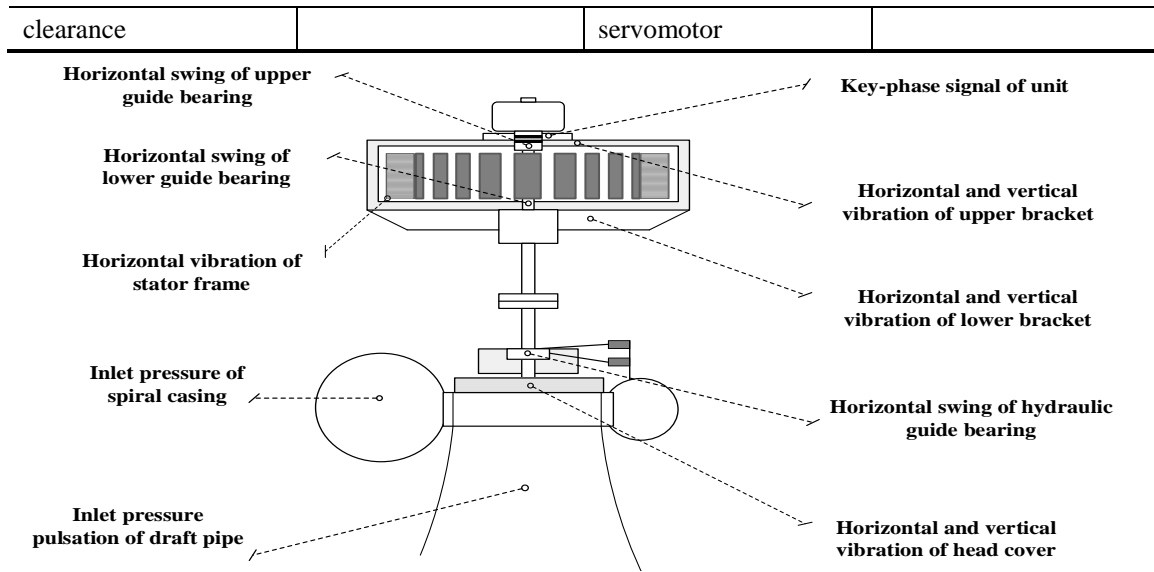
#### 4. Dynamic Balance Experiment on HGUs

In order to conduct a safety analysis on the HGU with load, a dynamic balance experiment was carried out on the HGU in an existing hydropower station in China and seventeen critical safety indices (i.e. X1-X17) were determined. These indices could

reflect the instability of the system with respect to vibrations and pressure pulsations in units. There are four Francis HGU's at the studied station, with installed and unit capacity of 1050MW and 262.5MW, respectively. In this experiment, the utilized sensors and measurement equipment for vibration analysis include: the PSTA-H vibration instrumentation of HGU, the TTS216 dynamic signal instrumentation of HGU, a CWY eddy current displacement sensor, a DP low-frequency vibration sensor, a KYB pressure transmitter and shielded signal cables. Some of the technical details of the four HGU's tested in the experiment are listed in Table 1, and the arrangements of the monitoring points on the HGU's, as well as the type of acquired data at each point, are presented in Fig. 4.

**Table 1** Information of the Francis hydraulic turbine of four HGU's in an existing hydropower station.

Information of Francis Hydraulic Turbines			
Type	HLS270-LJ-680	Nominal power	267.85MW
Nominal head	64m	Nominal flow	460.46m <sup>3</sup> /s
Nominal speed	93.75rpm	Runaway speed	185rpm
Number of runner blades	13	Number of movable guide vanes	24
Information of Generators			
Type	SF265-64/15000	Nominal capacity	291.7MVA
Stator voltage	15750V	Stator current	10692A
Power factor	0.9	Exciting voltage	350V
Exciting current	1900A	Nominal frequency	50Hz
Information of Governors			
Type	PFWT-200-6.3	Main configuration diameter	200mm
Operating oil pressure	6.3MPa	Servomotor stroke	780mm
Lower guide bearing clearance	0.15~0.2mm	Upper guide bearing clearance	0.15~0.2mm
Water guide bearing	0.2~0.25mm	Cylinder diameter of	640mm



**Fig. 4** Arrangements of monitoring points on HGU and type of recorded data at each point in

dynamic balance experiment in an existing hydropower station.

The initial running states of the four HGUs are different due to the internal coupled characteristics and external environment. A start-up test and a turbine-speed test are carried out for different HGUs before the dynamic balance experiments. This results in identifying the initial running state of the four HGUs, including that the rotating and fixed components for HGUs 1 and 4 operate normally and their vibration and swing values meet the design requirements. For HGUs 2 and 3, the start-up test shows that the rotating and fixed components run without abnormal friction or collision. Based on the turbine speed test at nominal speed for HGU 2, it is found that the horizontal vibration of upper bracket (290 $\mu\text{m}$ ), vertical vibration of upper bracket (157 $\mu\text{m}$ ), swing of upper guide bearing (335 $\mu\text{m}$ ), swing of lower guide bearing (417 $\mu\text{m}$ ) and swing of hydraulic guide bearing (382 $\mu\text{m}$ ) exceed the design requirements. Similarly for HGU 3, the horizontal



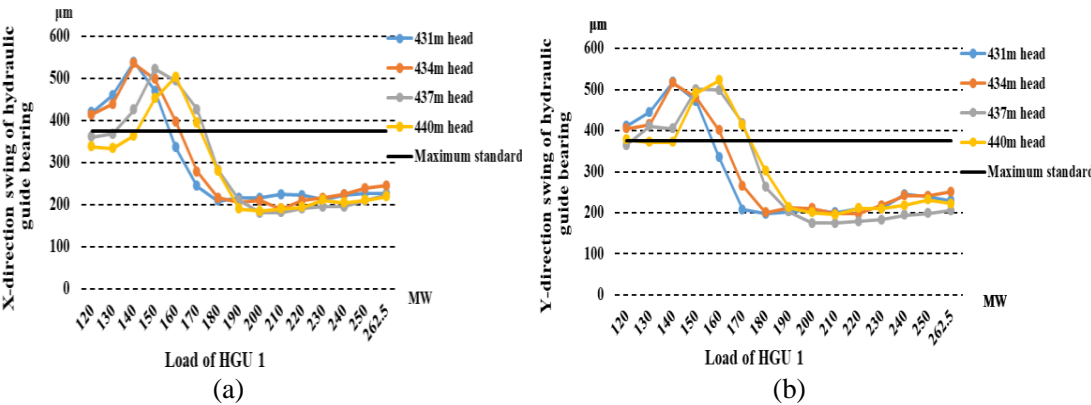
vibration of upper bracket (203 $\mu$ m) and swing of hydraulic guide bearing (657 $\mu$ m) exceed the design requirements. Moreover, the actual operating conditions for four HGUs with different allowable heads (431m, 434m, 437m and 440m) in experiment are listed in Table 2.

**Table 2** Actual operating conditions for four HGUs with different allowable heads (431m, 434m, 437m and 440m) used in the dynamic balance experiment.

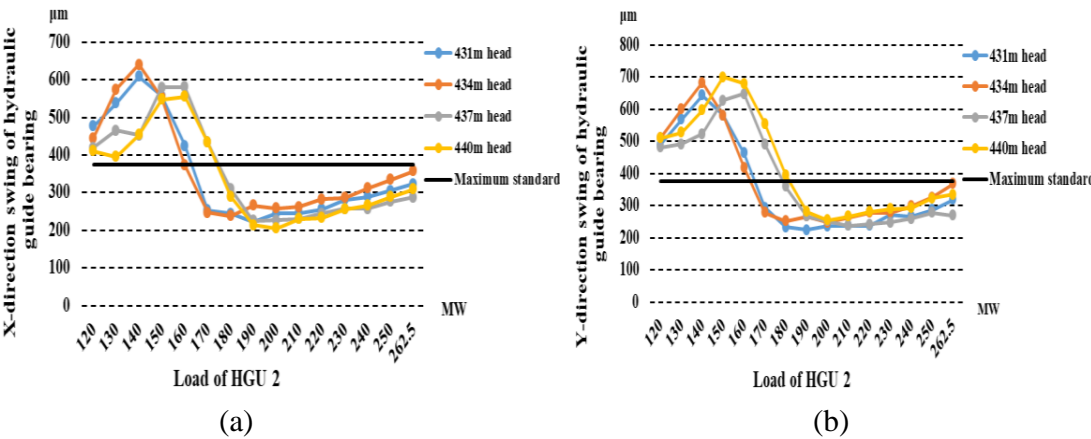
HGU 1			
	Actual upstream head	Actual downstream head	Actual head of station
431m Head	431.71m	366.64m	65.07m
434m Head	433.60m	366.36m	67.24m
437m Head	436.40m	366.24m	70.16m
440m Head	439.40m	367.98m	71.42m
HGU 2			
	Actual upstream head	Actual downstream head	Actual head of station
431m Head	431.92m	366.11m	65.81m
434m head	433.23m	365.62m	67.61m
437m head	437.33m	367.16m	70.17
440m head	439.60m	368.29m	71.31m
HGU 3			
	Actual upstream head	Actual downstream head	Actual head of station
431m head	431.93m	367.19m	64.74m
434m head	433.14m	366.27m	66.87m
437m head	437.14m	367.48m	69.66m
440m head	439.96m	367.87m	72.09m
HGU 4			
	Actual upstream head	Actual downstream head	Actual head of station
431m head	432.66m	367.38m	65.28m
434m head	433.31m	365.92m	67.39m
437m head	437.87m	367.97m	69.90m
440m head	439.60m	367.67m	71.93m

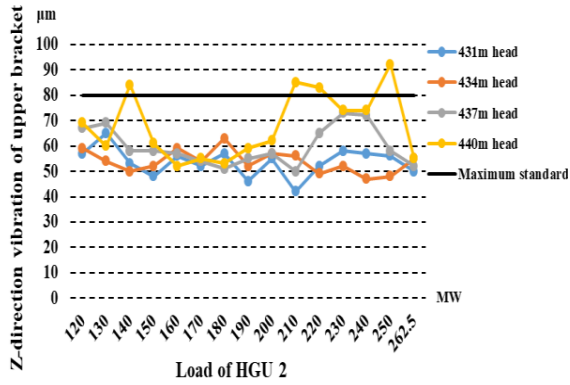
According to the design criteria, the operating head for the four HGUs in the studied station varies within the range of 431m to 440m. Four typical allowable heads (i.e. 431m, 434m, 437m and 440m) were chosen to conduct the dynamic balance experiment, where

vibration, swing and water pressure were measured. Based on the requirement of the actual operation in this station, the measurements were taken for various on-load conditions within the load range of 120MW to 265.2MW. The necessary indices in this experiment were selected to qualitatively investigate the stability of four HGU's, and the results are shown in Figs. 5 to 8.

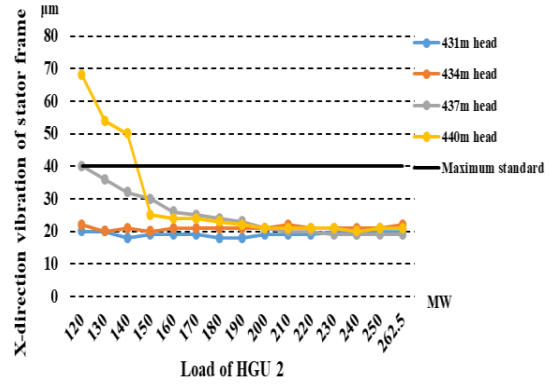


**Fig. 5** Measurements of vibration property in dynamic balance experiment of HGU 1 at an existing hydropower station, China.



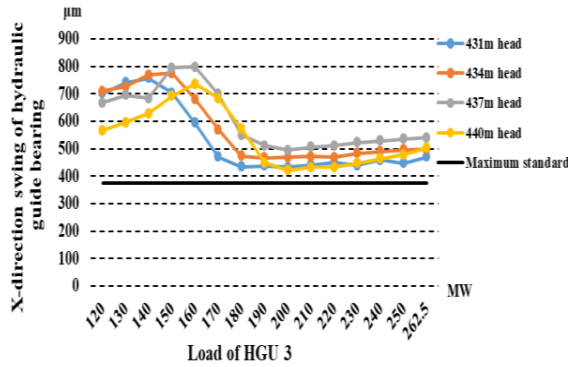


(c)

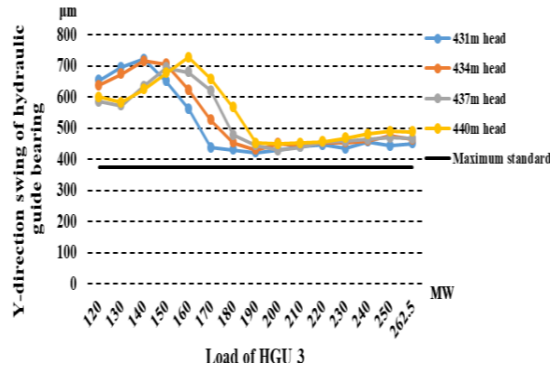


(d)

**Fig. 6** Measurements of vibration property in dynamic balance experiment of HGU 2 at an existing hydropower station, China.

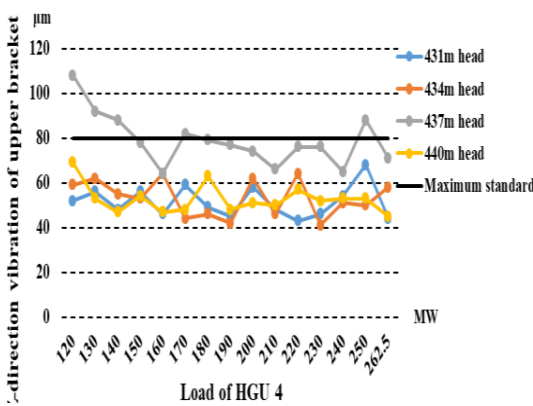


(a)

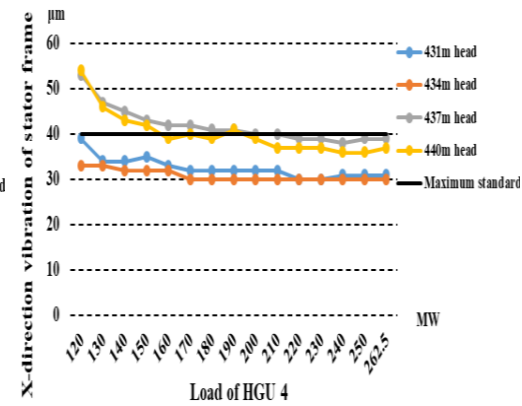


(b)

**Fig. 7** Measurements of vibration property in dynamic balance experiment of HGU 3 at an existing hydropower station, China.



(a)



(b)

**Fig. 8** Measurements of vibration property in dynamic balance experiment of HGU 4 at an existing hydropower station, China.

To evaluate the stability of each HGU, the measured vibrations at different points are compared with the maximum allowable vibration adopted from the national standards [43, 44]. The allowable range for all indices (X1-X17) are listed in Table 3.

**Table 3** Allowable ranges of HGU's indices (X1-X17) for safety operation from the national standards [43, 44].

Index (X1-X9)	Allowable range	Index (X10-X17)	Allowable range
Inlet pressure pulsation of draft pipe (X1)	0~64kPa	Z-direction vertical vibration of upper bracket (X10)	0~80μm
X-direction swing of upper guide bearing (X2)	0~300μm	X-direction horizontal vibration of lower bracket (X11)	0~110μm
Y-direction swing of upper guide bearing (X3)	0~300μm	Y-direction horizontal vibration of lower bracket (X12)	0~110μm
X-direction swing of lower guide bearing (X4)	0~300μm	Z-direction vertical vibration of lower bracket (X13)	0~80μm
Y-direction swing of lower guide bearing (X5)	0~300μm	X-direction vibration of stator frame (X14)	0~40μm
X-direction swing of hydraulic guide bearing (X6)	0~375μm	X-direction horizontal vibration of head cover (X15)	0~90μm
Y-direction swing of hydraulic guide bearing (X7)	0~375μm	Y-direction horizontal vibration of head cover (X16)	0~90μm
X-direction horizontal vibration of upper bracket (X8)	0~110μm	Z-direction vertical vibration of head cover (X17)	0~110μm
Y-direction horizontal vibration of upper bracket (X9)	0~110μm		

As illustrated in Table 3 and Figs. 5 to 8, each HGU has a level exceeding the allowable vibrations. Through a comparison of the results, it can be seen that the most stable HGU is unit 4 with the minimum vibration in the upper bracket (along Z-direction) and in its stator frame (along X-direction). It can be seen in Figs. 5 to 7, that the vibration of units 1, 2 and 3 are caused by two indices, i.e. swing of the hydraulic guide bearing along X and Y directions. However, it should be noted that the vibration magnitude of

these units is different where  $Y^3 > Y^2 > Y^1$  and  $X^3 > X^2 > X^1$  (e.g.  $Y^3$  and  $X^3$  refer to the magnitude of vibration in unit 3 along Y and X directions, respectively). The results of qualitative analysis highlight that the lowest level of safety among the studied units at the studied station is for unit 4, while unit 2 shows a more stable operation. Unit 1 has a higher safety level than unit 2, however, it does not provide an optimal condition. During the analysis of unit 3 responses, additional vibrations were observed in the upper bracket (along Z-direction) and the stator frame (along X-direction). Since it could not be determined, based on a qualitative assessment, to what extent the different indices affect the operational performance of the four HGU's, a rigorous quantitative analysis is required to investigate the safety condition of these four units.

## 5. Analysis of HGU's

In order to more effectively analyze the safety of the HGU's at the studied station, the grey correlation method is employed based on the results of dynamic balance experiments. For this purpose, maximum vibrations of the seventeen indices are firstly adopted from the experiment results, as listed in Table 4. The maximum vibration of selected index is considered as the assessment criteria in qualitative analysis, where the optimum level of safety is set as  $0\mu\text{m}$  due to the characteristic of inverse indices. Results of the grey correlation analysis for the four units are presented in Figs. 9 and 10.

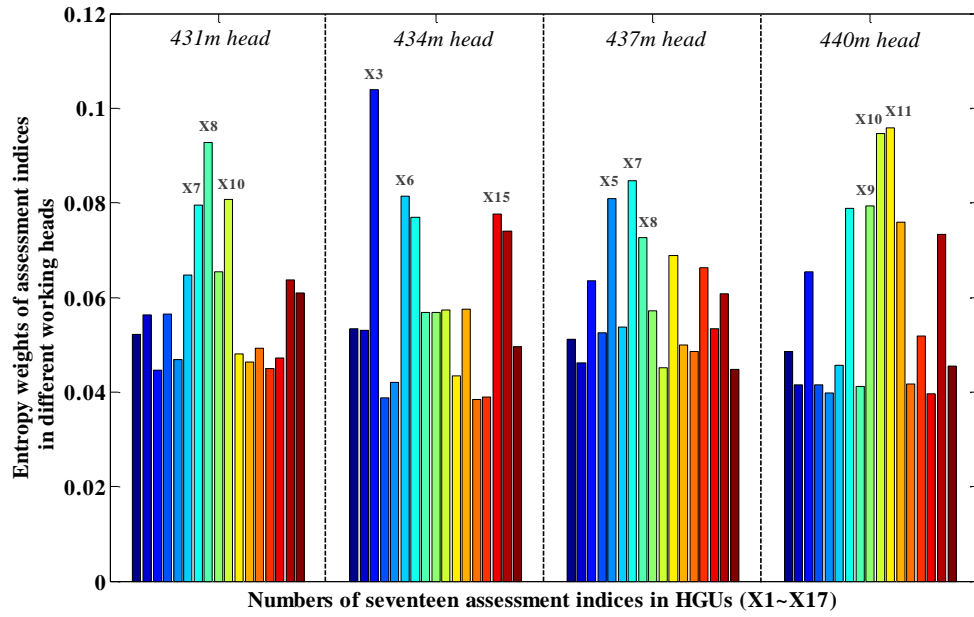
**Table 4** Measured Data: Maximum vibrations of seventeen assessment indices for HGU's (1-4) at an existing hydropower station, China.

Maximum vibrations ( $\mu\text{m}$ )
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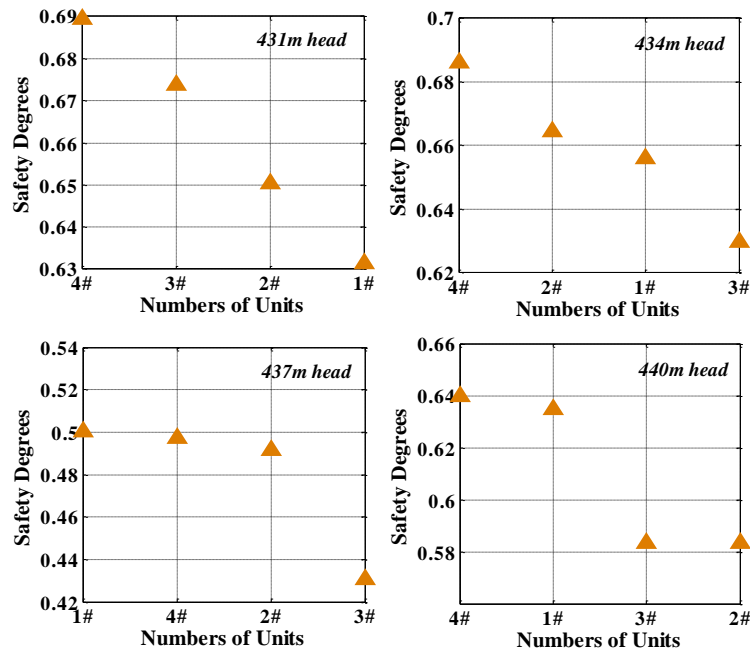
431m Head					434m Head			
Index	HGU 1	HGU 2	HGU 3	HGU 4	HGU 1	HGU 2	HGU 3	HGU 4
X1	32.69	62.94	36.55	49.24	48.73	72.58	70.05	82.23
X2	162	205	176	229	161	205	185	233
X3	160	249	164	168	158	258	193	244
X4	289	245	178	230	306	233	180	237
X5	328	241	209	196	340	234	203	280
X6	539	608	757	258	536	640	775	324
X7	519	643	721	234	516	682	716	288
X8	63	68	56	67	70	60	72	74
X9	77	66	73	60	70	56	60	64
X10	59	65	64	56	61	63	56	64
X11	28	17	17	11	36	14	18	25
X12	30	11	17	14	25	13	21	29
X13	56	62	41	88	59	56	58	163
X14	20	20	17	39	19	22	17	33
X15	30	37	26	27	40	31	56	41
X16	20	16	17	19	25	24	26	27
X17	61	27	44	75	53	56	59	76

**Maximum vibrations (µm)**

437m Head					440m Head			
Index	HGU 1	HGU 2	HGU 3	HGU 4	HGU 1	HGU 2	HGU 3	HGU 4
X1	69.89	61.19	95.52	79.04	86.67	168.14	121	46.39
X2	134	153	137	204	128	147	132	182
X3	141	195	151	214	151	210	162	201
X4	289	230	183	236	281	221	189	195
X5	252	186	131	237	289	157	180	178
X6	522	580	794	319	503	555	736	363
X7	501	648	694	290	523	700	727	365
X8	76	79	62	69	88	77	72	72
X9	92	70	67	106	98	96	64	71
X10	67	73	74	108	71	94	94	69
X11	25	97	82	29	26	19	25	25
X12	32	82	55	34	29	21	26	30
X13	76	15	255	115	68	108	185	102
X14	24	40	45	53	26	68	43	54
X15	82	63	107	48	63	94	61	66
X16	91	29	117	58	46	61	82	86
X17	92	79	306	90	81	109	140	74



**Fig. 9** Entropy weights of seventeen assessment indices for four on-load HGUs with different working heads.



**Fig. 10** Estimated safety levels of four on-load HGUs operating with different working heads at an existing hydropower station, China.

Fig. 9 indicates the assessment weights (i.e. the calculated entropy weights in Eq. (3)) of seventeen indices for HGUs operating with working heads of 431m, 434m, 437m and 440m. It should be noted that the same index assessed in different allowable heads has the same color. Considering Fig. 9, it is observed that the weight of each index differs considerably as the allowable head changes. This confirms the sensitivity of assessment indices on the HGUs' working heads as well as the fact that the information associated with the indices for the studied units is not identical. For instance, the highest weights for 431m working head are estimated as 0.093 for the horizontal vibration of upper bracket in X direction (X8 index), 0.081 for the vibration of upper bracket in Z direction (X10 index) and 0.08 for the swing of hydraulic guide bearing in Y direction (X7 index). Similarly, it is found that for the HGU with 434m working head, the main indices are X3, X6 and X15; for the 437m head unit, the main indices are X7, X5 and X8; and for the 440m head, they are X11, X10 and X9. Based on the effect of main indices and experimental results, the safety issues in the units with working heads of 431m, 434m and 437m may be caused by the integrating effect of mechanical problems and hydraulic imbalance while the mechanical component only results in a slight vibration of the units operating with the 440m head. It should also be noted that all assessment indices influence the safety of each unit although their contributions may vary significantly in different working heads.

Fig. 10 presents the estimated safety degree of the four HGUs under different working heads. The probabilistic results indicate that the most stable HGU is unit 4 with the average safety degree of 0.6282. Unit 1 is the second most stable unit with the



average safety degree of 0.6057. Unit 2 is the third safest unit of the four with the average safety degree of 0.5974 while unit 3 has the highest operational risk with its average safety degree of 0.5793. Based on the results, the system can safely run in the orders suggested in Fig. 10 when the allowable head fluctuates around 431m, 434m, 437m and 440m. However, when the hydropower station is not able to predict the working head of HGUs in advance, it is suggested that the optimal operational schedule is as follows: unit 4, unit 1, unit 2 and unit 3. This provides the safe operating strategy of HGUs to cope with peaks and troughs of electricity demand within the station.

It is also observed, in Fig. 10 that the safety degree of four units for the allowable head of 437m is lower than other working heads, changing between the range of [0.4305, 0.5004]. That is, the average safety of HUGs is less than 50 percent under the allowable head of 437m. It can therefore be a reasonable suggestion that the HGUs at the studied station could avoid, if possible, operating with this condition to enhance the operational safety.

## **6. Conclusions**

In this paper, a new framework is presented for the safety assessment of HGUs in hydropower stations and addresses the limitations in this research field. The study is carried out based on four on-load HGUs operating at an existing hydropower station in China. A dynamic balance experiment of the units with different allowable heads is conducted to qualitatively investigate the system stability and to obtain the requirements

for further quantitative analyses. This was performed by using the grey correlation analysis and entropy weights method. It is demonstrated that there is a significant difference in the sensitivity and risk contribution of the adopted indices between the allowable heads of 431m, 434m, 437m and 440m. The measurements of the weights reveal that, the safety of units operating with a head of 431m, 434m, 437m depend on the combined contribution of mechanical issues and hydraulic imbalance, while the undesired events occurring for units with 440m of head may only be caused by mechanical issues. From the grey-entropy assessment results, it can be concluded that the units have their specific safety degree as the allowable head changes. Moreover, a safe operational schedule can follow the order of: unit 4, unit 1, unit 2 and unit 3. It is anticipated that the proposed method can be adopted for improving the safety of hydropower facilities by providing optimal operational schedules.

## **Appendix**

### **Numerical process of the safety degree in HGUs**

The aim of the numerical analysis is to establish the grey-entropy correlation degree (see Eq. (7)) to conduct a dynamic safety assessment of on-load HGUs. Eq. (7) is combined with the entropy weights (see Eq. (3)) and the grey correlation coefficients (see Eq. (4)). That is, the numerical analysis consists of three steps to obtain the dynamic safety degree of HGUs: i) based on the measurement data of seventeen indices in Table 4, we calculate the

entropy weight matrix of index  $W_i$  with respect to different working heads, ii) estimating the correlation coefficient matrix of indices  $\xi_i(j)$  for different working heads based on the grey correlation equations (see Eqs. (4) to (6)) and iii) substituting the entropy weight matrix  $W_i$  and correlation coefficient matrix  $\xi_i(j)$  into the grey-entropy correlation degree (see Eq. (7)). Finally, the dynamic safety degree matrix of studied HGUs  $\alpha_i$  under different working heads is obtained. A detailed calculation progress is performed as follows.

In this study, we have seventeen assessment indices (marked as  $j$ ) and four HGUs (marked as  $i$ ) operating with four working heads of 431m, 434m, 437m and 440m. The optimum safety matrix is  $[0]$ , and the assessment matrices of the four HGUs at different working heads, i.e.  $[r_{ij}]_{431m}$ ,  $[r_{ij}]_{434m}$ ,  $[r_{ij}]_{437m}$ ,  $[r_{ij}]_{440m}$ , are shown in Table 4. The normalized method of inverse index expressed in Eq. (1) is used to obtain the standard form of optimum safety matrix and assessment matrices, which are

$$[0] \cap [r_{ij}]_{431m} =$$

$$\begin{bmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.4806 & 0.2926 & 0.3574 & 0 & 0 & 0.2880 & 0.2802 & 0.0735 & 0 & 0.0923 & 0 & 0 & 0.3636 & 0.4872 & 0.1892 & 0 & 0.1867 & 0 \\ 0 & 0.1048 & 0 & 0.1522 & 0.2652 & 0.1968 & 0.1082 & 0 & 0.1429 & 0 & 0.3929 & 0.6333 & 0.2955 & 0.4872 & 0 & 0.2000 & 0.6400 & 0 \\ 0.4193 & 0.2314 & 0.3414 & 0.3841 & 0.3628 & 0 & 0 & 0.1765 & 0.0519 & 0.0154 & 0.3929 & 0.4333 & 0.5341 & 0.5641 & 0.2973 & 0.1500 & 0.4133 & 0 \\ 0.2177 & 0 & 0.3253 & 0.2042 & 0.4024 & 0.6592 & 0.6755 & 0.0147 & 0.2208 & 0.1385 & 0.6071 & 0.5333 & 0 & 0 & 0.2703 & 0.0500 & 0 & 0 \end{bmatrix},$$

$$[0] \cap [r_{ij}]_{434m} =$$

$$\begin{bmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.4074 & 0.3090 & 0.3876 & 0 & 0 & 0.1625 & 0.2434 & 0.0541 & 0 & 0.0615 & 0 & 0.1379 & 0.6380 & 0.4242 & 0.0244 & 0.0741 & 0.3026 & 0 \\ 0.1174 & 0.1202 & 0 & 0.2386 & 0.3118 & 0 & 0 & 0.1892 & 0.2000 & 0.0308 & 0.6111 & 0.5517 & 0.6564 & 0.3333 & 0.2439 & 0.1111 & 0.2632 & 0 \\ 0 & 0 & 0.0543 & 0.2255 & 0.1765 & 0.4938 & 0.5777 & 0 & 0.0857 & 0.0154 & 0.3056 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2346 & 0.1202 & 0.0349 & 0.1993 & 0.2912 & 0.0500 & 0.0572 & 0.0811 & 0.0571 & 0 & 0.5278 & 0.6207 & 0.6196 & 0.3939 & 0.0976 & 0.4074 & 0.6447 & 0 \end{bmatrix},$$

$$[0] \cap [r_{ij}]_{437m} =$$

$$\begin{bmatrix}
1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\
0.2683 & 0.3431 & 0.3411 & 0 & 0 & 0.3426 & 0.2781 & 0.0380 & 0.1321 & 0.3796 & 0.7423 & 0.6098 & 0.7020 & 0.5472 & 0.2336 & 0.2222 & 0.6993 \\
0.3594 & 0.2500 & 0.0888 & 0.2042 & 0.2619 & 0.2695 & 0.0663 & 0 & 0.3396 & 0.3241 & 0 & 0 & 0.9412 & 0.2453 & 0.4112 & 0.7521 & 0.7418 \\
0 & 0.3284 & 0.2944 & 0.3668 & 0.4802 & 0 & 0 & 0.2152 & 0.3679 & 0.3148 & 0.1546 & 0.3293 & 0 & 0.1509 & 0 & 0 & 0 \\
0.1725 & 0 & 0 & 0.1834 & 0.0595 & 0.5982 & 0.5821 & 0.1266 & 0 & 0 & 0.7010 & 0.5854 & 0.5490 & 0 & 0.5514 & 0.5043 & 0.7059
\end{bmatrix}$$

and

$$[0] \cap [r_{ij}]_{440m} =$$

$$\begin{bmatrix}
1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\
0.4845 & 0.2967 & 0.2810 & 0 & 0 & 0.3166 & 0.2806 & 0 & 0 & 0.2447 & 0 & 0.0333 & 0.6324 & 0.6176 & 0.3298 & 0.4651 & 0.4214 \\
0 & 0.1923 & 0 & 0.2135 & 0.4567 & 0.2459 & 0.0371 & 0.1250 & 0.0204 & 0 & 0.2692 & 0.3000 & 0.4162 & 0 & 0 & 0.2907 & 0.2214 \\
0.2804 & 0.2747 & 0.2286 & 0.3274 & 0.3772 & 0 & 0 & 0.1818 & 0.3469 & 0 & 0.0385 & 0.1333 & 0 & 0.3676 & 0 & 0.0465 & 0 \\
0.7241 & 0 & 0.0429 & 0.3060 & 0.3841 & 0.5068 & 0.4979 & 0.1818 & 0.2755 & 0.2660 & 0.0385 & 0 & 0.4486 & 0.2059 & 0.3511 & 0 & 0.4714
\end{bmatrix}.$$

To clearly clarify the proposed method, an example for the assessment process of on-load HGU's at 440m working head is demonstrated as follows:

(i) **Entropy weight matrix  $W_i$** : Based on Eq. (2) and (3), the entropy weight matrix of seventeen indices derived from assessment matrix  $[r_{ij}]_{440m}$  is written as:

$$W_i = \begin{bmatrix}
0.0486 & 0.0415 & 0.0654 & 0.0415 & 0.0398 & 0.0456 & 0.0788 & 0.0412 \\
0.0793 & 0.0947 & 0.0959 & 0.0759 & 0.0417 & 0.0518 & 0.0396 & 0.0733 & 0.0455
\end{bmatrix}.$$

(ii) **Correlation coefficient matrix  $\xi_i(j)$** :

The minimum and maximum differences in the first level in Eq. (5) are obtained as:

$$\begin{cases}
\Delta_i \min = [0.2759 & 0.7033 & 0.7190 & 0.6726 & 0.5433 & 0.4932 & 0.5021 & 0.8182 \\
& 0.6531 & 0.7340 & 0.7308 & 0.7000 & 0.3676 & 0.3824 & 0.6489 & 0.5349 & 0.5286] \\
\Delta_i \max = [1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1]
\end{cases}$$

The minimum and maximum differences in the second level in Eq. (6) are obtained as:

$$\begin{cases}
\min_i(\Delta_i \min) = 0.2759 \\
\max_i(\Delta_i \max) = 1
\end{cases}$$

We substitute the obtained values for  $\Delta_i \min$ ,  $\Delta_i \max$ ,  $\min_i(\Delta_i \min)$  and

456  $\max(\Delta_i, \max)$  into Eq. (4), the correlation coefficient matrix,  $\xi_i(j)$ , between  $x_0$  and  $x_i$

457 with respect to the  $j$ th factor in the index set  $[r_{ij}]_{440m}$  is estimated as:

458  $\xi_i(j) =$

459 
$$\begin{bmatrix} 0.7641 & 0.6448 & 0.6365 & 0.5173 & 0.5173 & 0.6556 & 0.6363 & 0.5173 & 0.5173 & 0.6181 & 0.5173 & 0.5290 & 0.8943 & 0.8794 & 0.6630 & 0.7497 & 0.7194 \\ 0.5173 & 0.5933 & 0.5173 & 0.6031 & 0.7437 & 0.6187 & 0.5304 & 0.5643 & 0.5244 & 0.5173 & 0.6304 & 0.6466 & 0.7159 & 0.5173 & 0.5173 & 0.6416 & 0.6068 \\ 0.6362 & 0.6332 & 0.6103 & 0.6617 & 0.6910 & 0.5173 & 0.5173 & 0.5886 & 0.6729 & 0.5173 & 0.5309 & 0.5677 & 0.5173 & 0.6852 & 0.6753 & 0.5338 & 0.5173 \\ 1.0000 & 0.5173 & 0.5325 & 0.6499 & 0.6953 & 0.7812 & 0.7743 & 0.5886 & 0.6337 & 0.6287 & 0.5309 & 0.5173 & 0.7380 & 0.5996 & 0.6454 & 0.5173 & 0.7543 \end{bmatrix}.$$

460 **(iii) Grey-entropy correlation degree (also called safety degree)  $\alpha_{i_{440m}}$ :**

461 The grey-entropy correlation degree,  $\alpha_i$ , between the optimum unit and the studied

462 unit  $i$  can be estimated using Eq. (6). Thus, the safety degree matrix of the four HGUs at

463 the working head of 431m is

464 
$$\alpha_{i_{440m}} = \begin{bmatrix} 0.6350 \\ 0.5833 \\ 0.5834 \\ 0.6399 \end{bmatrix}, i=1, 2, 3 \text{ and } 4.$$

465 Similarly, we can obtain the safety degree matrices of the four HGUs at the working

466 head of 431m, 434m and 437m, respectively. The corresponding safety degree matrices

467 of the four HGUs are listed as follows:

468 431m working head:

469 
$$\alpha_{i_{431m}} = \begin{bmatrix} 0.6315 \\ 0.6504 \\ 0.6738 \\ 0.6895 \end{bmatrix}, i=1, 2, 3 \text{ and } 4.$$

470 434m working head:

$$\alpha_{i_{434m}} = \begin{bmatrix} 0.6560 \\ 0.6645 \\ 0.6296 \\ 0.6860 \end{bmatrix}, i=1, 2, 3 \text{ and } 4.$$

437m working head:

$$\alpha_{i_{437m}} = \begin{bmatrix} 0.5004 \\ 0.4915 \\ 0.4305 \\ 0.4974 \end{bmatrix}, i=1, 2, 3 \text{ and } 4.$$

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1     **Safety assessment of hydro-generating units using experiments**  
2                     **and grey-entropy correlation analysis**

3  
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20    **Abstract:** This paper focuses on the safety analysis of a nonlinear hydro-generating unit  
21    (HGU) running under different loads. For this purpose, a dynamic balance experiment  
22    implemented on an existing hydropower station in China is considered, to qualitatively  
23    investigate the stability of the system and to obtain the necessary indices for safety  
24    assessment. The experimental data are collected from four on-load units operating at  
25    different working heads including 431m, 434m, 437m, and 440m. A quantitative analysis  
26    on the safety performance of the four units was carried out by employing an integration of  
27    entropy weights method with grey correlation analysis. This assisted in obtaining the safety  
28    degree of each unit, providing the risk prompt to the operation of nonlinear

hydro-generating units. The results confirm that unit 4 has the highest level of safety while unit 3 operates with the lowest safety condition. This provides the optimal operational schedule of HGU's to cope with the fluctuations of electricity demand in the studied station. The proposed methodology in this paper is not only applicable to the HGU's in the studied station but could also be adopted to assess the safety degree of any hydropower facility.

**Keywords:** hydro-generating unit; dynamic balance experiment; safety analysis; grey-entropy correlation;

## 1. Introduction

Renewable energy is unarguably one of the most critical governing factors for today's increasing global economic and social development [1]. The pressing challenge lies in the sustainable harnessing of reliable, secure and affordable energy [2]. To date, hydropower has been the main renewable source of electrical energy for many countries' power consumption (e.g. 99% in Norway, 86% in Brazil and 76% in Switzerland) due to the environmental consequences of fossil fuels exploitation [3]. The electricity provided by hydropower contributes about 16% of the world total electricity generation and is expected to grow to 2 GW in thirty years [4]. It is therefore no exaggeration that hydropower represents more than 92% of generated green energy making it a significant contributor to the global electricity supply [5].

Hydropower stations are the major electricity generation facilities in which the hydro-generating unit (HGU) is the heart of the energy production, transmission and

conversion in each station [6]. HGU is a complex nonlinear system that integrates the characteristics of fluid, machinery, and electromagnetic induction [7]. A universal HGU is comprised of various coupled components such as hydraulic turbines, shafting systems, generators, governors, and excitation systems ([8] to [12]).

Due to the nonlinear coupled characteristics, several hazardous factors are present within the operation of an HGU including shafting vibrations, electromechanical delays, stochastic instability, and inefficient operation. A large number of literatures have extensively studied such topics from the perspective of individual subcomponents, which supports the research foundation for the safety study in this paper. For instance, literatures ([13], [14]) analyzed the cause of shafting vibrations in an HGU. Literature [15] studied a class of hydro-turbine with electromechanical delays. Researchers in ([16], [17]) modelled stochastic variables of an HGU to analyze its effect on the stability of subcomponents. Researchers in ([18], [19]) studied the adaptation strategy of hydropower systems to improve the operating efficiency. This range of conducted research highlights that the hydropower industry is greatly concerned about the safety of HGU operations and improvements are needed [20]. In particular, with the construction of large-capacity hydropower stations to be completed in the following decades, resolving the stability problems of operation, from the perspective of systemic properties, will be one of the major areas that attracts a great deal of attention from the industry [21]. Although a large number of advanced safety assessment methods have been developed in various research fields such as information science [22], ecological engineering [23] and marine engineering [24,

25], the operational safety of HGUs has been rarely investigated and very little evidence of achievements has been previously provided.

To date, the safety analyses of HGUs have mainly focused on investigating the stability of HGU components. The developed methods determine the instability status of the HGU components in terms of narrow hydraulic, mechanical, or electrical angle. However, the integrated safety level of the entire HGU system has not been evaluated from these independent components. Hence, there is a need for a framework that can assess the safety of HGU from the system perspective. Previous researches ([26] to [30]) developed a framework, combining the method of entropy weights and grey correlation theory to investigate the quality problems in different applications such as wastewater treatment, soil detection, and machinery fault. Several studies ([31], [32], and [33]) indicate that the method of entropy weights has a great potential for the assessment of complex systems by measuring the uncertainties of structure indices. The outcome of researches ([34], [35], and [36]) reveal that the grey correlation theory can be adopted for various prediction applications of such complex systems based on incomplete information.

The present paper herein investigates the operational stability of a nonlinear HGU and proposes a methodology for safety assessment of these systems. For this purpose, a dynamic balance experiment is conducted on four HGU units, each with a different working head, in an existing hydropower station in China. The experiment is based on vibration parameter, which is the main risk factor of on-load HGUs. Seventeen indices are extracted to qualitatively assess the operational stability of the units. An effective



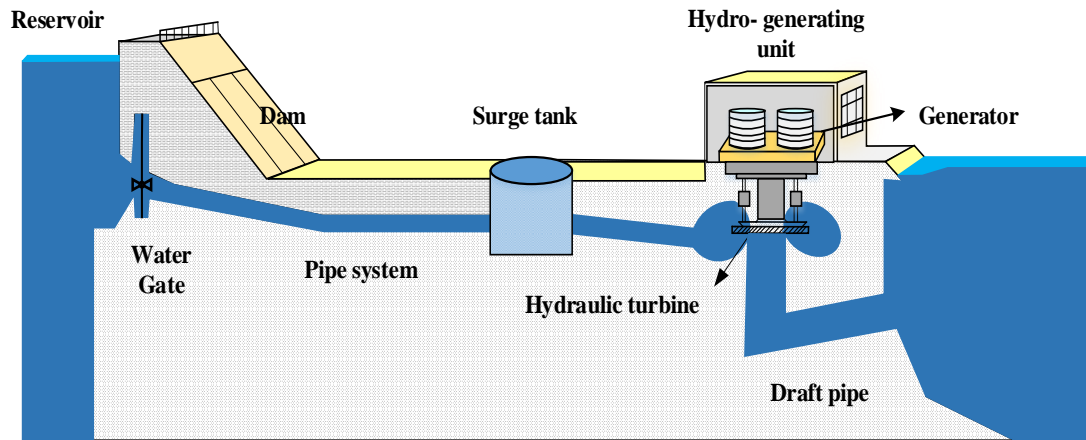
approach integrating the entropy theory and grey correlation is then utilized to quantitatively analyze the safety performance of the studied HGU. This assisted in determining the safety degree of the analyzed four units that run with load, as well as an optimal operational schedule of HGUs coping with peaks and troughs of electricity demand in the studied hydropower station.

The present paper has extensively reviewed the existing literature that are based on the individual subcomponents (e.g. hydro-turbines, shafts and generators) of HGU systems. The major contribution of the paper, however, is to consider the coupled characteristics of hydraulic, mechanical and electrical subcomponents for investigating the safety of HGU operation. Moreover, there are few researches that have successfully applied dynamic safety assessment to nonlinear HGUs. This paper presents a novel methodology that is significantly more flexible and efficient in dynamic safety assessment of HGUs with an attempt to overcome the limitations of static approaches. The safety degree of HGUs is quantified by using a probabilistic approach, which serves as a tool for monitoring and predicting the risk of accidents in hydropower stations resulting from failure in HGUs. This not only improves the safety of HGU operation, but also effectively reduces the operational and maintenance costs of energy production. The results obtained from this research benefit the operators and risk managers of the hydropower industry serving as a tool for development of risk mitigation strategies. For instance, it enables them to respond to the important question of “how to efficiently and safely arrange the operation of multiple HGUs with respect to different allowing heads”.

The remainder of the paper is structured as follows. In Section 2 a brief review of a universal nonlinear HGU is presented. In Section 3 the fundamentals of utilized methods and an overview of the global methodology for safety assessment of HGU are provided. Section 4 discusses the details of the conducted dynamic balance experiment on the studied station's HGU. Section 5 demonstrates the process of safety assessment methodology and presents its highlighted results. Lastly, the key findings of this study are discussed in the conclusion section.

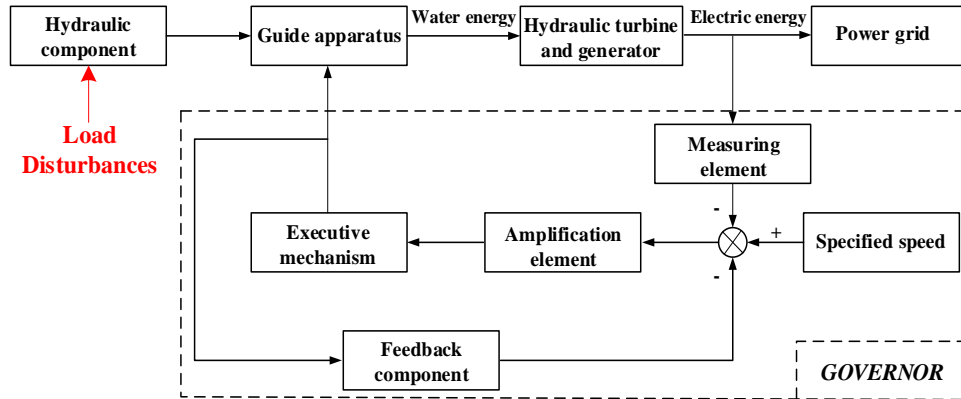
## **2. A Brief Review of an on-load HGU**

HGU is the key equipment of hydropower stations used to produce, transmit and converse electrical energy, which mainly consists of hydraulic turbines, generators, control systems/governors, excitation systems and inlet and draft pipes [37]. The operation of an HGU is always integrated with a number of other hydraulic components such as surge tank, piping system, water gate and reservoir [38]. The structure of an HGU and the key elements of the hydraulic system are shown in Fig. 1.



**Fig. 1** Schematic of an HGU.

HGU, in fact, is a nonlinear system with multi-attribute characteristics including hydraulic, mechanical, electrical and electromagnetic. An on-load HGU is a system synchronized with the power grid, and its load generally cannot be constantly maintained due to the stochastic load. The on-load HGU may be considered as a dynamic system varying with the changes (decrease or increase) in load. An HGU mainly utilizes pressure and momentum energy to produce power. The working mechanism of an on-load HGU is described as the flow velocity influenced by the effect of blade changes as the system load fluctuates, which in turn generates a reactive force in the flow channel. This drives the hydraulic turbines which generate mechanical energy, and the generator further converts the mechanical energy to electrical energy. The details of an HGU working mechanism is presented in Fig. 2.



**Fig. 2** Details of an on-load HGU working mechanism.

In actual hydropower stations, the dynamic performance of HGUs is hard to detect due to the rapid changes in the operational conditions influenced by internal couplings as well as the external environment. Uncontrolled and abrupt changes in the dynamic variables influencing the operational conditions of the system could result in critical damage to the asset as well as other consequences. It is therefore essential to conduct quantitative assessment of the safety and stability of an HGU, probably based on experimental investigations.

### 3. Methodology

Previous researches in this field have focused on developing static safety assessment frameworks for operating HGUs. However, due to the nonlinearity of these systems, attending to the dynamic effects in the analysis are essential for achieving better results. To overcome this shortcoming, an effective method must be developed applicable to hydropower facilities. Through conducting an interdisciplinary research [26, 27], this

section presents the details of an enhanced grey-entropy correlation methodology for dynamic safety analysis of on-load HGUs. The proposed framework is able to improve the imprecision of subjective entropy weights as well as the static evaluation of grey correlation degrees. A major contribution of the established method is in adopting the probabilistic approaches to predict and reflect the real-time safety level of on-load HGUs, which is greatly beneficial when dealing in a timely manner with unexpected accidents and the development of improved safety and risk mitigation strategies.

### **3.1 Entropy Weights Method**

The concept of entropy that is derived from thermodynamics theories represents a measure of disorder in a system. Entropy theory was proposed by Shannon, in 1948, to reflect the uncertainty in information science, it has been applied in various research fields for its precision and flexibility [39].

Two approaches can be applied for determining the weights of indices, known as subjective fixed weight and objective fixed weight methods. Entropy weight method, as an objective approach, is based on the amount of data, overcoming the subjectivity issues as it is independent of expert judgment. The main idea of entropy method is to determine the weights by index variations. In general, a smaller index weight represents a larger degree of index variation, meaning that the index may provide more assessment information and have significant influence on the stability of the system. In the entropy safety assessment of an HGU, a specific index weight is the critical indicator to measure the importance of the selected index, assessing its safety contribution to the studied

178 system.

179 Assuming that there are  $m$  assessment indices and  $n$  assessment units, the assessment  
180 data is transformed into a form of standardization that employs a normalized method of  
181 inverse index, shown in Eq. (1) [40].

$$182 \quad r_{ij} = \frac{\max x_{ij} - x_{ij}}{\max x_{ij} - \min x_{ij}}, \quad i=1,2,\dots,m \text{ and } j=1,2,\dots,n, \quad (1)$$

183 where  $\{r_{ij}\}_{m \times n}$  is the normalized set of inverse index.  $\max x_{ij}$  and  $\min x_{ij}$  are the  
184 maximum and minimum values in the index column of assessment units, respectively. It  
185 should be noted that the lower value of inverse index is most important in ensuring safe  
186 operation of an HGU.

187 Then the entropy value of index  $i$  is determined by Eq. (2).

$$188 \quad E_i = -\frac{\sum_{j=1}^n r_{ij} \ln r_{ij}}{\ln n}, \quad i=1,2,\dots,m \quad (2)$$

189 and the index weight of  $i$  is obtained as:

$$190 \quad \omega_i = \frac{1 - E_i}{\sum_{i=1}^m (1 - E_i)}, \quad \sum_{j=1}^n \omega_i = 1, \quad \omega_i \in [0,1] \quad (3)$$

191 Therefore, the index weight set  $W_i$  is  $[\omega_1, \omega_2, \dots, \omega_n]$ .

### 192 3.2 Grey-entropy Correlation Method

193 Grey system is used to describe an uncertain system that has the characteristic of  
194 partial information loss, and grey correlation theory is a powerful tool to query the quality  
195 of a system with poor information [41]. An on-load HGU is an engineering system

196 incorporating a degree of uncertainty and therefore it can be assessed by the grey  
 197 correlation theory. The concept of using grey theory is to find the possible motion rule  
 198 from the disordered and fuzzy data. Specifically, it is the similarity of an index in  
 199 different assessment units that is the key factor for measuring the variation between the  
 200 indices. A greater similarity between indices means that the grey correlation of a studied  
 201 unit is more optimal. There are no requirements for the size and characteristics of data in  
 202 a grey correlation analysis which overcomes the shortcomings of traditional regression  
 203 analyses.

204 Based on the normalized set of inverse index  $\{r_{ij}\}_{m \times n}$  mentioned in Eq. (1), the  
 205 index column is expressed as  $x_1, x_2, \dots, x_m$ . It should be noted that, there are  $i$  assessment  
 206 plans in the analysis, i.e.,  $x_i = [x_i(1), x_i(2), \dots, x_i(n)]$ , where  $x_0$  is assumed to be the  
 207 optimum plan. Therefore, the correlation coefficient,  $\xi_i(j)$ , between  $x_0$  and  $x_i$  with  
 208 respect to the  $j^{\text{th}}$  factor in the index set  $\{r_{ij}\}_{m \times n}$  is expressed as [42]:

$$209 \quad \xi_i(j) = \frac{\min_i(\Delta_i \min) + \rho \max_i(\Delta_i \max)}{\Delta_i + \rho \max_i(\Delta_i \max)}, \quad i=1,2,\dots,m \text{ and } j=1,2,\dots,n, \quad (4)$$

210 where  $\Delta_i$  is equal to  $|x_0(j) - x_i(j)|$ ,  $\rho$  is the resolution coefficient that changes  
 211 within the interval  $[0, 1]$ , but generally it is set at 0.5.  $\Delta_i \min$  and  $\Delta_i \max$  denote the  
 212 minimum and maximum differences in the first level respectively, while  $\min_i(\Delta_i \min)$   
 213 and  $\max_i(\Delta_i \max)$  are the minimum and maximum differences in the second level,  
 214 respectively. The expressions for each of these terms are shown as follows:

$$\begin{cases} \Delta_i \min = \min_j |x_0(j) - x_i(j)| \\ \Delta_i \max = \max_j |x_0(j) - x_i(j)| \end{cases} \quad (5)$$

and

$$\begin{cases} \min_i(\Delta_i \min) = \min_i \min_j |x_0(j) - x_i(j)| \\ \max_i(\Delta_i \max) = \max_i \max_j |x_0(j) - x_i(j)| \end{cases}, \quad (6)$$

Subsequently, based on the index weight  $W_i$  obtained using Eq. (3), we estimate the correlation coefficient  $\xi_i(j)$  for the  $i^{\text{th}}$  studied unit to obtain its integrating safety degree. Therefore, the grey correlation degree,  $\alpha_i$ , between the optimum unit and the studied unit  $i$  is given by the grey-entropy correlation equation as follows:

$$\alpha_i = \sum_{j=1}^m W_i \xi_i(j), \quad 0 \leq \alpha_i \leq 1. \quad (7)$$

In Eq. (7), the obtained grey correlation degree  $\alpha_i$ , also defined as the safety degree, assists in assessing the safety level of a multi-unit HGU from a probabilistic point of view. That is, a higher value of  $\alpha_i$  corresponds to a safer HGU thus for instance, a system with  $\alpha_i=1$  has the maximum level of reliability.

### 3.3 Global Methodology

This paper presents a novel framework for the dynamic safety assessment of HGUs by combining the entropy weight method with the grey correlation analysis. The major novel components of the proposed method consist of:- firstly, the method overcomes the subjectivity of traditional methods in determining the weight coefficients of assessment indices, which improves the accuracy of the results and provides a more scientific



representation. Secondly, the method completely transforms the static safety assessment into a dynamic practice by substituting the dynamic entropy weights (i.e. Eq. (3)) into the relationship for obtaining the grey correlation degree (i.e. Eq. (7)). Thirdly, few existing studies have been proven to be successful in conducting a probabilistic safety analysis of nonlinear HGUs.

The steps of the developed methodology in this paper are provided in Fig. 3, and summarized as follows.

(1) A dynamic balance experiment is carried out on the existing HGUs for different allowing heads, to qualitatively analyze the dynamic operational behavior of a hydropower station. The obtained data,  $m$  assessment indices for  $n$  studied HGUs, is later used to conduct a quantitative safety analysis.

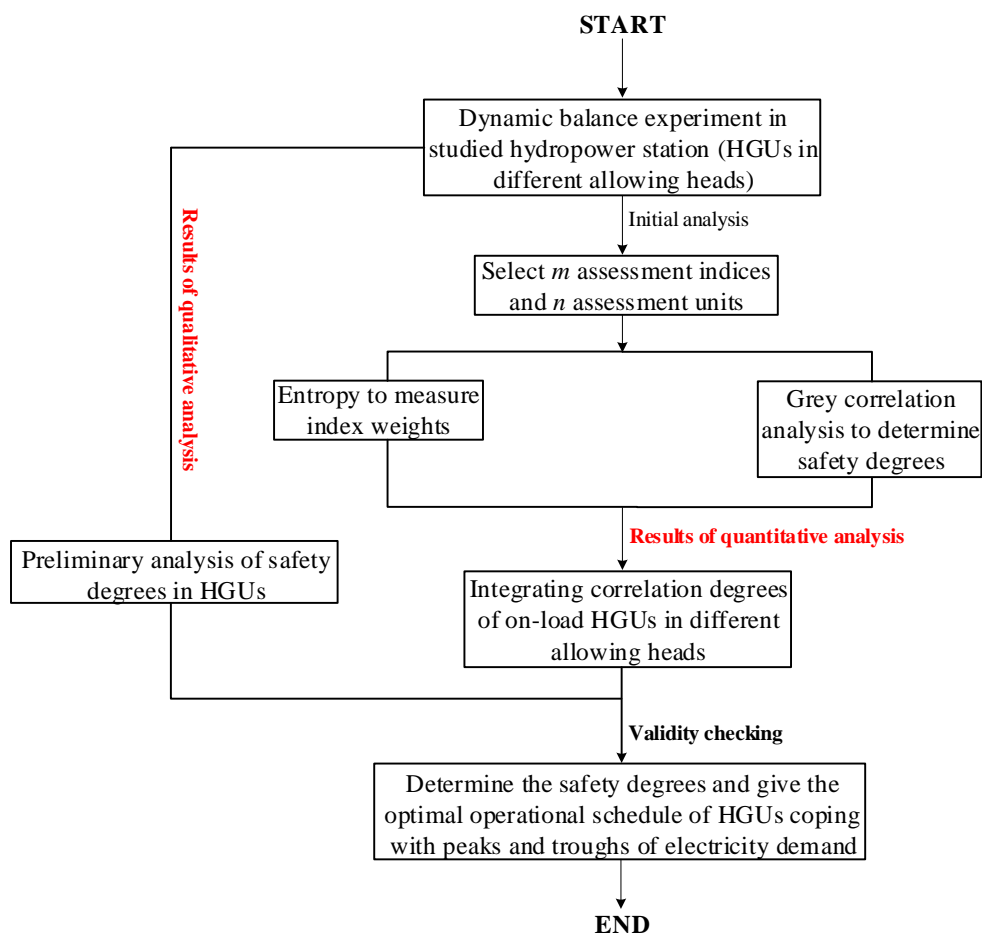
(2) Dynamic entropy weights (see Eq. (3)) are developed to estimate the contribution of the indices on HGSSs' stability with respect to time. For this purpose, the indices with significant influence on HGS' operation under various allowing heads are identified.

(3) The grey-entropy correlation degrees (see Eq. (7)), combined with the dynamic entropy weights (see Eq. (3)) and grey correlation coefficients (see Eq. (4)), are used to evaluate the safety degree of  $n$  studied HGUs. The safety degree is expressed by probability values.

(4) Based on the quantitative analysis, the time-varying safety state of HGUs and any accidents are revealed. This enables the technicians and operators of hydropower stations to make an optimal operational schedule of HGUs for dealing with fluctuations of

electricity generation and demand.

A detailed illustration of the numerical process of entropy weights and safety degrees is presented in the Appendix.



**Fig. 3** Proposed framework for safety assessment of on-load HGUs.

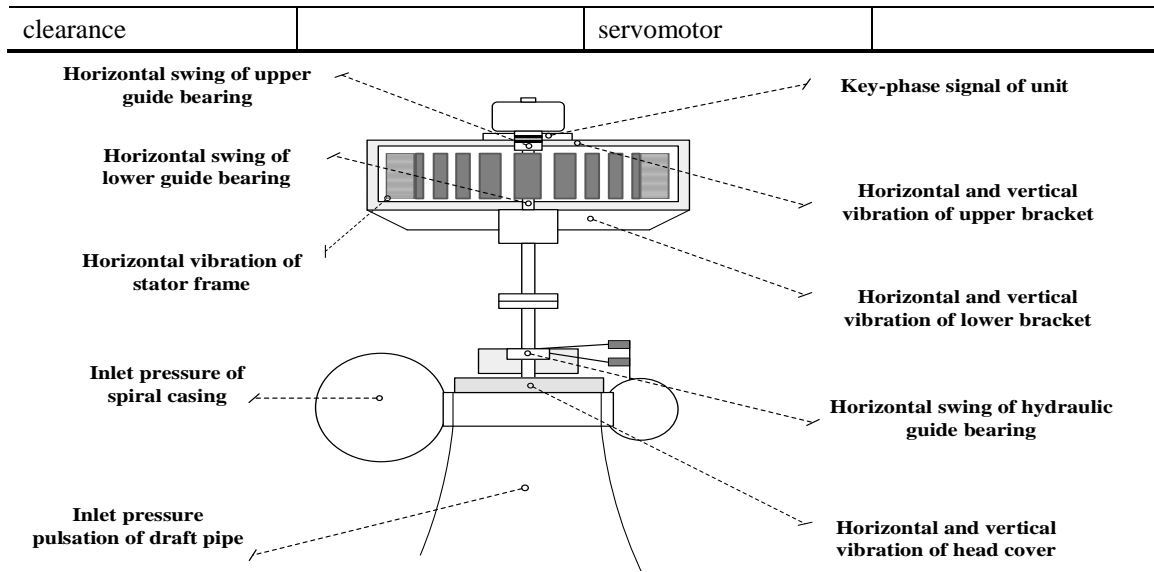
#### 4. Dynamic Balance Experiment on HGUs

In order to conduct a safety analysis on the HGU with load, a dynamic balance experiment was carried out on the HGU in an existing hydropower station in China and seventeen critical safety indices (i.e. X1-X17) were determined. These indices could

reflect the instability of the system with respect to vibrations and pressure pulsations in units. There are four Francis HGU's at the studied station, with installed and unit capacity of 1050MW and 262.5MW, respectively. In this experiment, the utilized sensors and measurement equipment for vibration analysis include: the PSTA-H vibration instrumentation of HGU, the TTS216 dynamic signal instrumentation of HGU, a CWY eddy current displacement sensor, a DP low-frequency vibration sensor, a KYB pressure transmitter and shielded signal cables. Some of the technical details of the four HGU's tested in the experiment are listed in Table 1, and the arrangements of the monitoring points on the HGU's, as well as the type of acquired data at each point, are presented in Fig. 4.

**Table 1** Information of the Francis hydraulic turbine of four HGU's in an existing hydropower station.

Information of Francis Hydraulic Turbines			
Type	HLS270-LJ-680	Nominal power	267.85MW
Nominal head	64m	Nominal flow	460.46m <sup>3</sup> /s
Nominal speed	93.75rpm	Runaway speed	185rpm
Number of runner blades	13	Number of movable guide vanes	24
Information of Generators			
Type	SF265-64/15000	Nominal capacity	291.7MVA
Stator voltage	15750V	Stator current	10692A
Power factor	0.9	Exciting voltage	350V
Exciting current	1900A	Nominal frequency	50Hz
Information of Governors			
Type	PFWT-200-6.3	Main configuration diameter	200mm
Operating oil pressure	6.3MPa	Servomotor stroke	780mm
Lower guide bearing clearance	0.15~0.2mm	Upper guide bearing clearance	0.15~0.2mm
Water guide bearing	0.2~0.25mm	Cylinder diameter of	640mm



**Fig. 4** Arrangements of monitoring points on HGU and type of recorded data at each point in

dynamic balance experiment in an existing hydropower station.

The initial running states of the four HGUs are different due to the internal coupled characteristics and external environment. A start-up test and a turbine-speed test are carried out for different HGUs before the dynamic balance experiments. This results in identifying the initial running state of the four HGUs, including that the rotating and fixed components for HGUs 1 and 4 operate normally and their vibration and swing values meet the design requirements. For HGUs 2 and 3, the start-up test shows that the rotating and fixed components run without abnormal friction or collision. Based on the turbine speed test at nominal speed for HGU 2, it is found that the horizontal vibration of upper bracket (290 $\mu\text{m}$ ), vertical vibration of upper bracket (157 $\mu\text{m}$ ), swing of upper guide bearing (335 $\mu\text{m}$ ), swing of lower guide bearing (417 $\mu\text{m}$ ) and swing of hydraulic guide bearing (382 $\mu\text{m}$ ) exceed the design requirements. Similarly for HGU 3, the horizontal

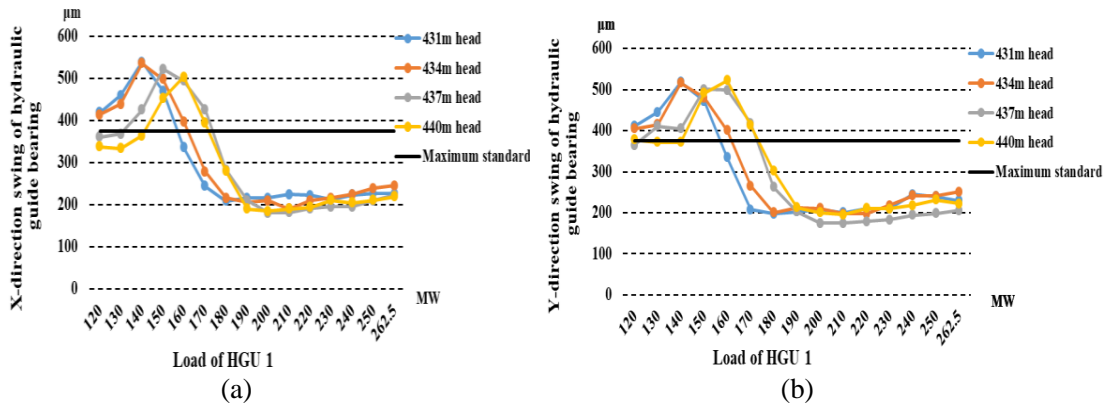
vibration of upper bracket (203 $\mu$ m) and swing of hydraulic guide bearing (657 $\mu$ m) exceed the design requirements. Moreover, the actual operating conditions for four HGUs with different allowable heads (431m, 434m, 437m and 440m) in experiment are listed in Table 2.

**Table 2** Actual operating conditions for four HGUs with different allowable heads (431m, 434m, 437m and 440m) used in the dynamic balance experiment.

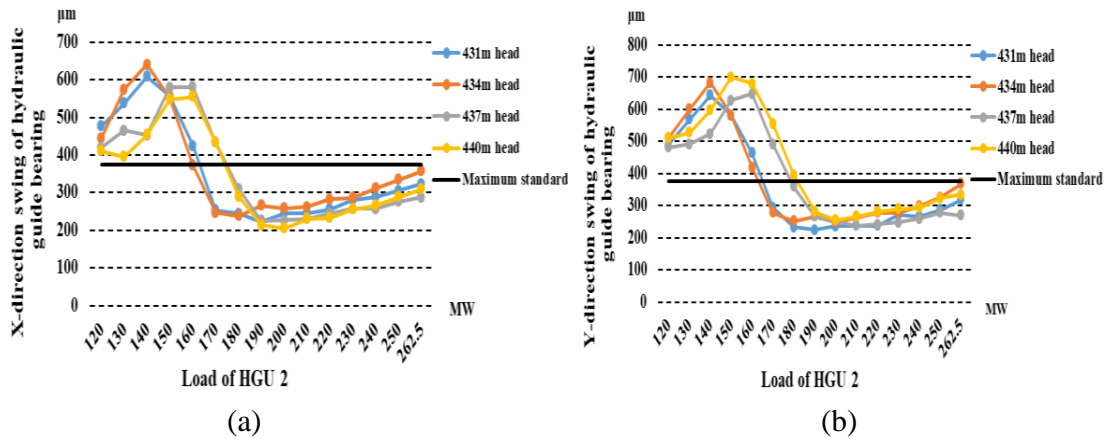
HGU 1			
	Actual upstream head	Actual downstream head	Actual head of station
431m Head	431.71m	366.64m	65.07m
434m Head	433.60m	366.36m	67.24m
437m Head	436.40m	366.24m	70.16m
440m Head	439.40m	367.98m	71.42m
HGU 2			
	Actual upstream head	Actual downstream head	Actual head of station
431m Head	431.92m	366.11m	65.81m
434m head	433.23m	365.62m	67.61m
437m head	437.33m	367.16m	70.17
440m head	439.60m	368.29m	71.31m
HGU 3			
	Actual upstream head	Actual downstream head	Actual head of station
431m head	431.93m	367.19m	64.74m
434m head	433.14m	366.27m	66.87m
437m head	437.14m	367.48m	69.66m
440m head	439.96m	367.87m	72.09m
HGU 4			
	Actual upstream head	Actual downstream head	Actual head of station
431m head	432.66m	367.38m	65.28m
434m head	433.31m	365.92m	67.39m
437m head	437.87m	367.97m	69.90m
440m head	439.60m	367.67m	71.93m

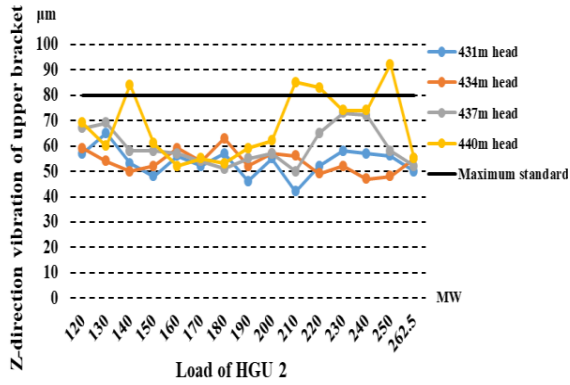
According to the design criteria, the operating head for the four HGUs in the studied station varies within the range of 431m to 440m. Four typical allowable heads (i.e. 431m, 434m, 437m and 440m) were chosen to conduct the dynamic balance experiment, where

vibration, swing and water pressure were measured. Based on the requirement of the actual operation in this station, the measurements were taken for various on-load conditions within the load range of 120MW to 265.2MW. The necessary indices in this experiment were selected to qualitatively investigate the stability of four HGU's, and the results are shown in Figs. 5 to 8.

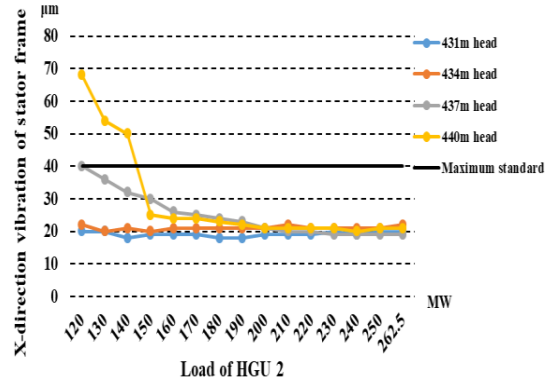


**Fig. 5** Measurements of vibration property in dynamic balance experiment of HGU 1 at an existing hydropower station, China.



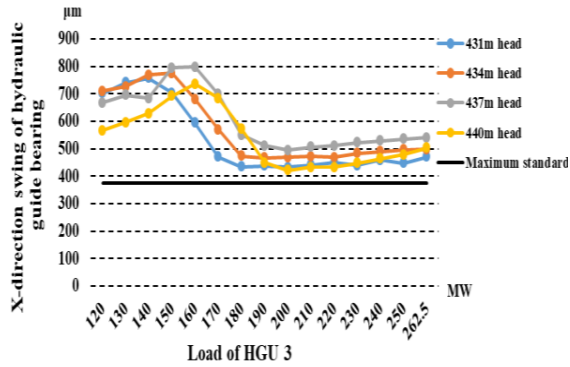


(c)

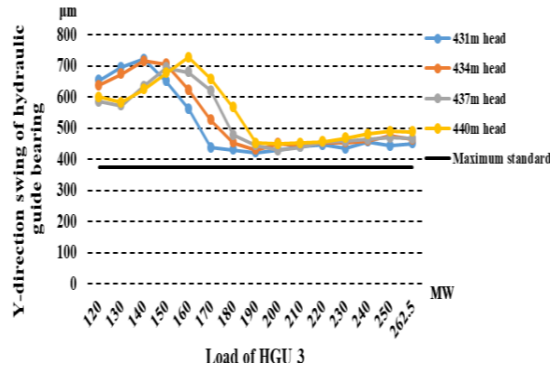


(d)

**Fig. 6** Measurements of vibration property in dynamic balance experiment of HGU 2 at an existing hydropower station, China.

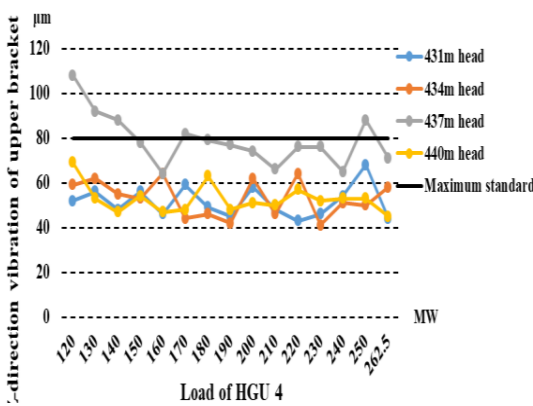


(a)

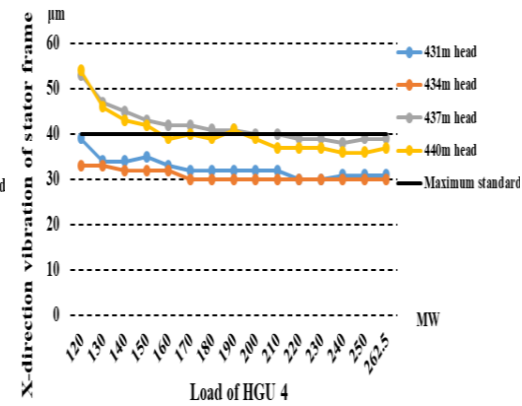


(b)

**Fig. 7** Measurements of vibration property in dynamic balance experiment of HGU 3 at an existing hydropower station, China.



(a)



(b)

**Fig. 8** Measurements of vibration property in dynamic balance experiment of HGU 4 at an existing hydropower station, China.

To evaluate the stability of each HGU, the measured vibrations at different points are compared with the maximum allowable vibration adopted from the national standards [43, 44]. The allowable range for all indices (X1-X17) are listed in Table 3.

**Table 3** Allowable ranges of HGU's indices (X1-X17) for safety operation from the national standards [43, 44].

Index (X1-X9)	Allowable range	Index (X10-X17)	Allowable range
Inlet pressure pulsation of draft pipe (X1)	0~64kPa	Z-direction vertical vibration of upper bracket (X10)	0~80μm
X-direction swing of upper guide bearing (X2)	0~300μm	X-direction horizontal vibration of lower bracket (X11)	0~110μm
Y-direction swing of upper guide bearing (X3)	0~300μm	Y-direction horizontal vibration of lower bracket (X12)	0~110μm
X-direction swing of lower guide bearing (X4)	0~300μm	Z-direction vertical vibration of lower bracket (X13)	0~80μm
Y-direction swing of lower guide bearing (X5)	0~300μm	X-direction vibration of stator frame (X14)	0~40μm
X-direction swing of hydraulic guide bearing (X6)	0~375μm	X-direction horizontal vibration of head cover (X15)	0~90μm
Y-direction swing of hydraulic guide bearing (X7)	0~375μm	Y-direction horizontal vibration of head cover (X16)	0~90μm
X-direction horizontal vibration of upper bracket (X8)	0~110μm	Z-direction vertical vibration of head cover (X17)	0~110μm
Y-direction horizontal vibration of upper bracket (X9)	0~110μm		

As illustrated in Table 3 and Figs. 5 to 8, each HGU has a level exceeding the allowable vibrations. Through a comparison of the results, it can be seen that the most stable HGU is unit 4 with the minimum vibration in the upper bracket (along Z-direction) and in its stator frame (along X-direction). It can be seen in Figs. 5 to 7, that the vibration of units 1, 2 and 3 are caused by two indices, i.e. swing of the hydraulic guide bearing along X and Y directions. However, it should be noted that the vibration magnitude of



these units is different where  $Y^3 > Y^2 > Y^1$  and  $X^3 > X^2 > X^1$  (e.g.  $Y^3$  and  $X^3$  refer to the magnitude of vibration in unit 3 along Y and X directions, respectively). The results of qualitative analysis highlight that the lowest level of safety among the studied units at the studied station is for unit 4, while unit 2 shows a more stable operation. Unit 1 has a higher safety level than unit 2, however, it does not provide an optimal condition. During the analysis of unit 3 responses, additional vibrations were observed in the upper bracket (along Z-direction) and the stator frame (along X-direction). Since it could not be determined, based on a qualitative assessment, to what extent the different indices affect the operational performance of the four HGU's, a rigorous quantitative analysis is required to investigate the safety condition of these four units.

## 5. Analysis of HGU's

In order to more effectively analyze the safety of the HGU's at the studied station, the grey correlation method is employed based on the results of dynamic balance experiments. For this purpose, maximum vibrations of the seventeen indices are firstly adopted from the experiment results, as listed in Table 4. The maximum vibration of selected index is considered as the assessment criteria in qualitative analysis, where the optimum level of safety is set as  $0\mu\text{m}$  due to the characteristic of inverse indices. Results of the grey correlation analysis for the four units are presented in Figs. 9 and 10.

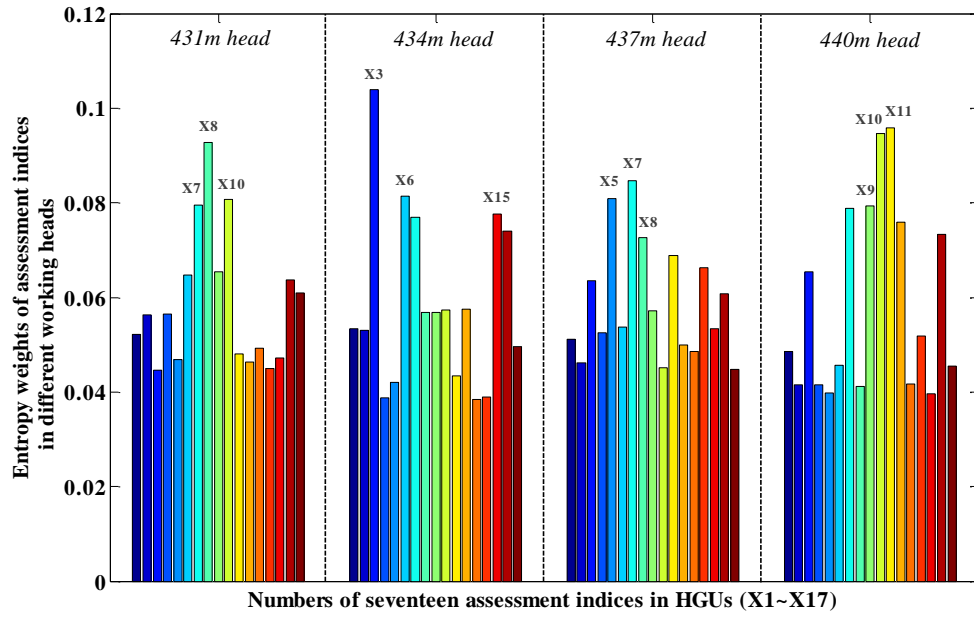
**Table 4** Measured Data: Maximum vibrations of seventeen assessment indices for HGU's (1-4) at an existing hydropower station, China.

Maximum vibrations ( $\mu\text{m}$ )
--------------------------------------

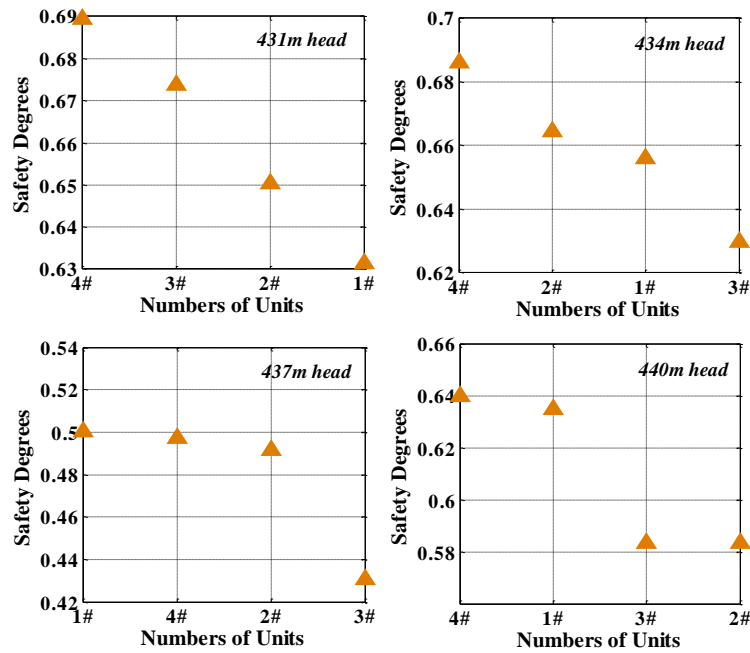
431m Head					434m Head			
Index	HGU 1	HGU 2	HGU 3	HGU 4	HGU 1	HGU 2	HGU 3	HGU 4
X1	32.69	62.94	36.55	49.24	48.73	72.58	70.05	82.23
X2	162	205	176	229	161	205	185	233
X3	160	249	164	168	158	258	193	244
X4	289	245	178	230	306	233	180	237
X5	328	241	209	196	340	234	203	280
X6	539	608	757	258	536	640	775	324
X7	519	643	721	234	516	682	716	288
X8	63	68	56	67	70	60	72	74
X9	77	66	73	60	70	56	60	64
X10	59	65	64	56	61	63	56	64
X11	28	17	17	11	36	14	18	25
X12	30	11	17	14	25	13	21	29
X13	56	62	41	88	59	56	58	163
X14	20	20	17	39	19	22	17	33
X15	30	37	26	27	40	31	56	41
X16	20	16	17	19	25	24	26	27
X17	61	27	44	75	53	56	59	76

**Maximum vibrations (µm)**

437m Head					440m Head			
Index	HGU 1	HGU 2	HGU 3	HGU 4	HGU 1	HGU 2	HGU 3	HGU 4
X1	69.89	61.19	95.52	79.04	86.67	168.14	121	46.39
X2	134	153	137	204	128	147	132	182
X3	141	195	151	214	151	210	162	201
X4	289	230	183	236	281	221	189	195
X5	252	186	131	237	289	157	180	178
X6	522	580	794	319	503	555	736	363
X7	501	648	694	290	523	700	727	365
X8	76	79	62	69	88	77	72	72
X9	92	70	67	106	98	96	64	71
X10	67	73	74	108	71	94	94	69
X11	25	97	82	29	26	19	25	25
X12	32	82	55	34	29	21	26	30
X13	76	15	255	115	68	108	185	102
X14	24	40	45	53	26	68	43	54
X15	82	63	107	48	63	94	61	66
X16	91	29	117	58	46	61	82	86
X17	92	79	306	90	81	109	140	74



**Fig. 9** Entropy weights of seventeen assessment indices for four on-load HGUs with different working heads.



**Fig. 10** Estimated safety levels of four on-load HGUs operating with different working heads at an existing hydropower station, China.

Fig. 9 indicates the assessment weights (i.e. the calculated entropy weights in Eq. (3)) of seventeen indices for HGUs operating with working heads of 431m, 434m, 437m and 440m. It should be noted that the same index assessed in different allowable heads has the same color. Considering Fig. 9, it is observed that the weight of each index differs considerably as the allowable head changes. This confirms the sensitivity of assessment indices on the HGUs' working heads as well as the fact that the information associated with the indices for the studied units is not identical. For instance, the highest weights for 431m working head are estimated as 0.093 for the horizontal vibration of upper bracket in X direction (X8 index), 0.081 for the vibration of upper bracket in Z direction (X10 index) and 0.08 for the swing of hydraulic guide bearing in Y direction (X7 index). Similarly, it is found that for the HGU with 434m working head, the main indices are X3, X6 and X15; for the 437m head unit, the main indices are X7, X5 and X8; and for the 440m head, they are X11, X10 and X9. Based on the effect of main indices and experimental results, the safety issues in the units with working heads of 431m, 434m and 437m may be caused by the integrating effect of mechanical problems and hydraulic imbalance while the mechanical component only results in a slight vibration of the units operating with the 440m head. It should also be noted that all assessment indices influence the safety of each unit although their contributions may vary significantly in different working heads.

Fig. 10 presents the estimated safety degree of the four HGUs under different working heads. The probabilistic results indicate that the most stable HGU is unit 4 with the average safety degree of 0.6282. Unit 1 is the second most stable unit with the

average safety degree of 0.6057. Unit 2 is the third safest unit of the four with the average safety degree of 0.5974 while unit 3 has the highest operational risk with its average safety degree of 0.5793. Based on the results, the system can safely run in the orders suggested in Fig. 10 when the allowable head fluctuates around 431m, 434m, 437m and 440m. However, when the hydropower station is not able to predict the working head of HGUs in advance, it is suggested that the optimal operational schedule is as follows: unit 4, unit 1, unit 2 and unit 3. This provides the safe operating strategy of HGUs to cope with peaks and troughs of electricity demand within the station.

It is also observed, in Fig. 10 that the safety degree of four units for the allowable head of 437m is lower than other working heads, changing between the range of [0.4305, 0.5004]. That is, the average safety of HUGs is less than 50 percent under the allowable head of 437m. It can therefore be a reasonable suggestion that the HGUs at the studied station could avoid, if possible, operating with this condition to enhance the operational safety.

## **6. Conclusions**

In this paper, a new framework is presented for the safety assessment of HGUs in hydropower stations and addresses the limitations in this research field. The study is carried out based on four on-load HGUs operating at an existing hydropower station in China. A dynamic balance experiment of the units with different allowable heads is conducted to qualitatively investigate the system stability and to obtain the requirements

for further quantitative analyses. This was performed by using the grey correlation analysis and entropy weights method. It is demonstrated that there is a significant difference in the sensitivity and risk contribution of the adopted indices between the allowable heads of 431m, 434m, 437m and 440m. The measurements of the weights reveal that, the safety of units operating with a head of 431m, 434m, 437m depend on the combined contribution of mechanical issues and hydraulic imbalance, while the undesired events occurring for units with 440m of head may only be caused by mechanical issues. From the grey-entropy assessment results, it can be concluded that the units have their specific safety degree as the allowable head changes. Moreover, a safe operational schedule can follow the order of: unit 4, unit 1, unit 2 and unit 3. It is anticipated that the proposed method can be adopted for improving the safety of hydropower facilities by providing optimal operational schedules.

## **Appendix**

### **Numerical process of the safety degree in HGUs**

The aim of the numerical analysis is to establish the grey-entropy correlation degree (see Eq. (7)) to conduct a dynamic safety assessment of on-load HGUs. Eq. (7) is combined with the entropy weights (see Eq. (3)) and the grey correlation coefficients (see Eq. (4)). That is, the numerical analysis consists of three steps to obtain the dynamic safety degree of HGUs: i) based on the measurement data of seventeen indices in Table 4, we calculate the

422 entropy weight matrix of index  $W_i$  with respect to different working heads, ii) estimating  
 423 the correlation coefficient matrix of indices  $\xi_i(j)$  for different working heads based on the  
 424 grey correlation equations (see Eqs. (4) to (6)) and iii) substituting the entropy weight  
 425 matrix  $W_i$  and correlation coefficient matrix  $\xi_i(j)$  into the grey-entropy correlation  
 426 degree (see Eq. (7)). Finally, the dynamic safety degree matrix of studied HGUs  $\alpha_i$  under  
 427 different working heads is obtained. A detailed calculation progress is performed as  
 428 follows.

429 In this study, we have seventeen assessment indices (marked as  $j$ ) and four HGUs  
 430 (marked as  $i$ ) operating with four working heads of 431m, 434m, 437m and 440m. The  
 431 optimum safety matrix is  $[0]$ , and the assessment matrices of the four HGUs at different  
 432 working heads, i.e.  $[r_{ij}]_{431m}$ ,  $[r_{ij}]_{434m}$ ,  $[r_{ij}]_{437m}$ ,  $[r_{ij}]_{440m}$ , are shown in Table 4. The  
 433 normalized method of inverse index expressed in Eq. (1) is used to obtain the standard  
 434 form of optimum safety matrix and assessment matrices, which are

435  $[0] \cap [r_{ij}]_{431m} =$

436 
$$\begin{bmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.4806 & 0.2926 & 0.3574 & 0 & 0 & 0.2880 & 0.2802 & 0.0735 & 0 & 0.0923 & 0 & 0 & 0.3636 & 0.4872 & 0.1892 & 0 & 0.1867 & 0 \\ 0 & 0.1048 & 0 & 0.1522 & 0.2652 & 0.1968 & 0.1082 & 0 & 0.1429 & 0 & 0.3929 & 0.6333 & 0.2955 & 0.4872 & 0 & 0.2000 & 0.6400 & 0 \\ 0.4193 & 0.2314 & 0.3414 & 0.3841 & 0.3628 & 0 & 0 & 0.1765 & 0.0519 & 0.0154 & 0.3929 & 0.4333 & 0.5341 & 0.5641 & 0.2973 & 0.1500 & 0.4133 & 0 \\ 0.2177 & 0 & 0.3253 & 0.2042 & 0.4024 & 0.6592 & 0.6755 & 0.0147 & 0.2208 & 0.1385 & 0.6071 & 0.5333 & 0 & 0 & 0.2703 & 0.0500 & 0 & 0 \end{bmatrix},$$

437  $[0] \cap [r_{ij}]_{434m} =$

438 
$$\begin{bmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.4074 & 0.3090 & 0.3876 & 0 & 0 & 0.1625 & 0.2434 & 0.0541 & 0 & 0.0615 & 0 & 0.1379 & 0.6380 & 0.4242 & 0.0244 & 0.0741 & 0.3026 & 0 \\ 0.1174 & 0.1202 & 0 & 0.2386 & 0.3118 & 0 & 0 & 0.1892 & 0.2000 & 0.0308 & 0.6111 & 0.5517 & 0.6564 & 0.3333 & 0.2439 & 0.1111 & 0.2632 & 0 \\ 0 & 0 & 0.0543 & 0.2255 & 0.1765 & 0.4938 & 0.5777 & 0 & 0.0857 & 0.0154 & 0.3056 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2346 & 0.1202 & 0.0349 & 0.1993 & 0.2912 & 0.0500 & 0.0572 & 0.0811 & 0.0571 & 0 & 0.5278 & 0.6207 & 0.6196 & 0.3939 & 0.0976 & 0.4074 & 0.6447 & 0 \end{bmatrix},$$

439  $[0] \cap [r_{ij}]_{437m} =$

$$\begin{bmatrix}
1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\
0.2683 & 0.3431 & 0.3411 & 0 & 0 & 0.3426 & 0.2781 & 0.0380 & 0.1321 & 0.3796 & 0.7423 & 0.6098 & 0.7020 & 0.5472 & 0.2336 & 0.2222 & 0.6993 \\
0.3594 & 0.2500 & 0.0888 & 0.2042 & 0.2619 & 0.2695 & 0.0663 & 0 & 0.3396 & 0.3241 & 0 & 0 & 0.9412 & 0.2453 & 0.4112 & 0.7521 & 0.7418 \\
0 & 0.3284 & 0.2944 & 0.3668 & 0.4802 & 0 & 0 & 0.2152 & 0.3679 & 0.3148 & 0.1546 & 0.3293 & 0 & 0.1509 & 0 & 0 & 0 \\
0.1725 & 0 & 0 & 0.1834 & 0.0595 & 0.5982 & 0.5821 & 0.1266 & 0 & 0 & 0.7010 & 0.5854 & 0.5490 & 0 & 0.5514 & 0.5043 & 0.7059
\end{bmatrix}$$

and

$$[0] \cap [r_{ij}]_{440m} =$$

$$\begin{bmatrix}
1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\
0.4845 & 0.2967 & 0.2810 & 0 & 0 & 0.3166 & 0.2806 & 0 & 0 & 0.2447 & 0 & 0.0333 & 0.6324 & 0.6176 & 0.3298 & 0.4651 & 0.4214 \\
0 & 0.1923 & 0 & 0.2135 & 0.4567 & 0.2459 & 0.0371 & 0.1250 & 0.0204 & 0 & 0.2692 & 0.3000 & 0.4162 & 0 & 0 & 0.2907 & 0.2214 \\
0.2804 & 0.2747 & 0.2286 & 0.3274 & 0.3772 & 0 & 0 & 0.1818 & 0.3469 & 0 & 0.0385 & 0.1333 & 0 & 0.3676 & 0 & 0.0465 & 0 \\
0.7241 & 0 & 0.0429 & 0.3060 & 0.3841 & 0.5068 & 0.4979 & 0.1818 & 0.2755 & 0.2660 & 0.0385 & 0 & 0.4486 & 0.2059 & 0.3511 & 0 & 0.4714
\end{bmatrix}.$$

To clearly clarify the proposed method, an example for the assessment process of on-load HGU's at 440m working head is demonstrated as follows:

(i) **Entropy weight matrix  $W_i$** : Based on Eq. (2) and (3), the entropy weight matrix of seventeen indices derived from assessment matrix  $[r_{ij}]_{440m}$  is written as:

$$W_i = \begin{bmatrix} 0.0486 & 0.0415 & 0.0654 & 0.0415 & 0.0398 & 0.0456 & 0.0788 & 0.0412 \\ 0.0793 & 0.0947 & 0.0959 & 0.0759 & 0.0417 & 0.0518 & 0.0396 & 0.0733 & 0.0455 \end{bmatrix}.$$

(ii) **Correlation coefficient matrix  $\xi_i(j)$** :

The minimum and maximum differences in the first level in Eq. (5) are obtained as:

$$\begin{cases} \Delta_i \min = [0.2759 & 0.7033 & 0.7190 & 0.6726 & 0.5433 & 0.4932 & 0.5021 & 0.8182 \\ & 0.6531 & 0.7340 & 0.7308 & 0.7000 & 0.3676 & 0.3824 & 0.6489 & 0.5349 & 0.5286] \\ \Delta_i \max = [1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1] \end{cases}$$

The minimum and maximum differences in the second level in Eq. (6) are obtained as:

$$\begin{cases} \min(\Delta_i \min) = 0.2759 \\ \max(\Delta_i \max) = 1 \end{cases}.$$

We substitute the obtained values for  $\Delta_i \min$ ,  $\Delta_i \max$ ,  $\min(\Delta_i \min)$  and



456  $\max(\Delta_i, \max)$  into Eq. (4), the correlation coefficient matrix,  $\xi_i(j)$ , between  $x_0$  and  $x_i$

457 with respect to the  $j$ th factor in the index set  $[r_{ij}]_{440m}$  is estimated as:

458  $\xi_i(j) =$

459 
$$\begin{bmatrix} 0.7641 & 0.6448 & 0.6365 & 0.5173 & 0.5173 & 0.6556 & 0.6363 & 0.5173 & 0.5173 & 0.6181 & 0.5173 & 0.5290 & 0.8943 & 0.8794 & 0.6630 & 0.7497 & 0.7194 \\ 0.5173 & 0.5933 & 0.5173 & 0.6031 & 0.7437 & 0.6187 & 0.5304 & 0.5643 & 0.5244 & 0.5173 & 0.6304 & 0.6466 & 0.7159 & 0.5173 & 0.5173 & 0.6416 & 0.6068 \\ 0.6362 & 0.6332 & 0.6103 & 0.6617 & 0.6910 & 0.5173 & 0.5173 & 0.5886 & 0.6729 & 0.5173 & 0.5309 & 0.5677 & 0.5173 & 0.6852 & 0.6753 & 0.5338 & 0.5173 \\ 1.0000 & 0.5173 & 0.5325 & 0.6499 & 0.6953 & 0.7812 & 0.7743 & 0.5886 & 0.6337 & 0.6287 & 0.5309 & 0.5173 & 0.7380 & 0.5996 & 0.6454 & 0.5173 & 0.7543 \end{bmatrix}.$$

460 **(iii) Grey-entropy correlation degree (also called safety degree)  $\alpha_{i_{440m}}$ :**

461 The grey-entropy correlation degree,  $\alpha_i$ , between the optimum unit and the studied

462 unit  $i$  can be estimated using Eq. (6). Thus, the safety degree matrix of the four HGUs at

463 the working head of 431m is

464 
$$\alpha_{i_{440m}} = \begin{bmatrix} 0.6350 \\ 0.5833 \\ 0.5834 \\ 0.6399 \end{bmatrix}, i=1, 2, 3 \text{ and } 4.$$

465 Similarly, we can obtain the safety degree matrices of the four HGUs at the working

466 head of 431m, 434m and 437m, respectively. The corresponding safety degree matrices

467 of the four HGUs are listed as follows:

468 431m working head:

469 
$$\alpha_{i_{431m}} = \begin{bmatrix} 0.6315 \\ 0.6504 \\ 0.6738 \\ 0.6895 \end{bmatrix}, i=1, 2, 3 \text{ and } 4.$$

470 434m working head:

$$\alpha_{i_{434m}} = \begin{bmatrix} 0.6560 \\ 0.6645 \\ 0.6296 \\ 0.6860 \end{bmatrix}, i=1, 2, 3 \text{ and } 4.$$

437m working head:

$$\alpha_{i_{437m}} = \begin{bmatrix} 0.5004 \\ 0.4915 \\ 0.4305 \\ 0.4974 \end{bmatrix}, i=1, 2, 3 \text{ and } 4.$$

474

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482

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